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FUZZY IDENTIFICATION AND CONTROL ALGORITHMS BASED ON AN ETSK MODEL[1]

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Abstract: This paper presents an extended Takagi-Sugeno-Kang Model (ETSK) based on extension principle. Its analytic expression is derived and an algorithm to identify such a model is proposed. A Variable Weights TSK Model (VWTSK) which is equivalent to ETSK model is induced, and an ETSK-model-based fuzzy control algorithm is presented. In this algorithm the fuzzy control rules are designed according to the rules of the VWTSK model. Simulation shows that an ETSK model can give out more accurate long-range predictions and the control algorithm can achieve better control performance than fuzzy PID control method. *Copyright © 1999 IFAC*

Keywords: Fuzzy Model; Fuzzy Control; Identification Algorithm; Fuzzy Logic; Fuzzy Inputs; Fuzzy Outputs; Rules.

1. INTRODUCTION

Fuzzy logic is widely applied to system modeling and control to deal with the relationship between imprecise, qualitative, linguistic assessments of the system's input and output states. Linguistic models (LM) (Sugeno, *et al.*, 1993) and Takagi-Sugeno-Kang model (TSK) (Sugeno, *et al.*, 1993; Takagi, *et al.*,1985) are two kinds of fuzzy models most commonly used.

For a Single Input Single Output (SISO) discrete time system in State-Space description

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k) = g(x(k), u(k)) \end{cases}$$

its Linguistic model is composed of rules in form of state space

IF $x_1(k)$ is A_{i1} AND,..., AND $x_p(k)$ is A_{ip}
AND $u(k)$ is B_{i0}
THEN $x_1(k+1)$ is C_{i1} AND,...,AND
$x_p(k+1)$ is C_{ip} AND $y(k)$ is D_i, i=1,...,m,

or in form of input-output

IF $y(k\text{-}1)$ is A_{i1} AND,..., AND $y(k\text{-}p)$ is A_{ip}
AND $u(k)$ is B_{i0} AND $u(k\text{-}1)$ is B_{i1}
AND,...,AND $u(k\text{-}q)$ is B_{iq}
THEN $y(k)$ is D_i, i=1,...,m

where $x(k)=[x_1(k),x_2(k),\cdots,x_p(k)]^T \in R^p$ is system state variables; $u(k)$ and $y(k)$ are input and output of the system; $x_j(k)$, $u(k\text{-}t)$, $y(k\text{-}j) \in R^1$; A_{ij}, B_{it}, C_{ij} and D_i are fuzzy subsets of the corresponding universe of discourse; j=1,$\cdots$, p, t=0,$\cdots$,q.

In a TSK model, instead of being a linguistic term, the consequent of the rule is a linear numerical function. The TSK model of the above system is composed of rules in the form

IF $y(k\text{-}1)$ is A_{i1} AND,..., AND $y(k\text{-}p)$ is A_{ip}
AND $u(k)$ is B_{i0} AND $u(k\text{-}1)$ is B_{i1}
AND, ..., AND $u(k\text{-}q)$ is B_{iq}

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THEN $y(k) = a_{i1}\, y(k-1) +,\ldots,+ a_{ip}\, y(k-p)$
$+ b_{i0}\, u(k) +,\ldots,+ b_{iq} u(k-q)$, $i=1,\ldots,m$
where $u(k)$ and $y(k)$ are input and output of the system at k time; a_{ij}, b_{it}, $y(k-j)$, $u(k-t) \in R^1$; A_{ij} and B_{it} are fuzzy input sets of the corresponding universe of discourse; $j=1,\cdots,p$, $t=0,\cdots,q$.

The fuzzy input sets of a TSK model provide a non-linear scheduling of the linear parameters, and the output is the synthesis of the linear functions, which only have a local influence.

2. ETSK MODEL AND ITS ANALYTIC EXPRESSION

As an extension of TSK model, an ETSK model for a SISO system is composed of rules in the form

IF $y(k-1)$ is A_{i1} AND,...,AND $y(k-p)$ is A_{ip}
AND $u(k)$ is B_{i0} AND $u(k-1)$ is B_{i1}
AND, ..., AND $u(k-q)$ is B_{iq}
THEN $Y = a_{i1} \otimes y(k-1) \oplus,\ldots,\oplus\, a_{ip} \otimes y(k-p)$
$\oplus\, b_{i0}\, u(k) \oplus,\ldots,\oplus\, b_{iq} \otimes u(k-q)$ (1)

where $u(k-j)$ and $y(k-t) \in R^1$ are the inputs of ETSK model; Y is the fuzzy output of ETSK model; a_{ij}, and b_{it} are LR (Left and Right reference) type fuzzy numbers (Dubois, *et al.*1980); A_{ij} and B_{it} are fuzzy input sets of the corresponding universe of discourse, $i=1,\ldots,m$, $j=1,\cdots,p$, $t=1,\cdots,q$; $\oplus$ is the extended addition operation of fuzzy numbers and $\otimes$ is the extended scalar product of fuzzy number.

Comparing with a TSK model, the consequent of the rule in an ETSK model is a fuzzy linear function, which is the combination of addition and scalar product of fuzzy numbers. There now follows a theorem relative to ETSK model.

Theorem 1 Given a fuzzy system F: $R^{p+q+1} \rightarrow R^1$ described by Eq. (1), if a_{ij} and b_{it} ($i=1,\cdots,m$, $j=1,\cdots,p$, $t=0,\cdots,q$) are LR type fuzzy numbers then the system's input-output relation is

$$y(k) = \mathrm{Centroid}(Y) = \frac{\sum_{i=1}^{m} \alpha_i (|\boldsymbol{h}|^{\tau} \boldsymbol{S}_i)(\boldsymbol{h}^{\tau} \boldsymbol{G}_i)}{\sum_{i=1}^{m} \alpha_i (|\boldsymbol{h}|^{\tau} \boldsymbol{S}_i)} \qquad (2)$$

where τ denotes the transpose, $y(k)$ is output of the system which is centroid of the fuzzy output Y,

$$\alpha_i = \prod_{j=1}^{p} \mu_{A_{ij}}(y(k-j)) \prod_{t=0}^{q} \mu_{B_{it}}(u(k-t)) \qquad (3)$$

indicates the degree the ith rule is activated,

$\boldsymbol{h}=[y(k-1),\ldots, y(k-p), u(k), u(k-1), \ldots, u(k-q)]^{\tau}$
is the input vector,

$|\boldsymbol{h}|=[|y(k-1)|,\ldots,|y(k-p)|,|u(k)|,|u(k-)|, \ldots ,|u(k-q)|]^{\tau}$
is defined,

$$s_{a_{ij}} = \int_{-\infty}^{\infty} \mu_{a_{ij}}(t)dt, \quad s_{b_{it}} = \int_{-\infty}^{\infty} \mu_{b_{it}}(t)dt$$

are the area enclosed by the membership function and the axis,

$$g_{a_{ij}} = \frac{\int_{-\infty}^{\infty} t\mu_{a_{ij}}(t)dt}{s_{a_{ij}}}, \quad g_{b_{it}} = \frac{\int_{-\infty}^{\infty} t\mu_{b_{it}}(t)dt}{s_{b_{it}}}$$

Is the projection of the gravity center on the axis,

$$\boldsymbol{S}_i = \left[s_{a_{i1}}, \ldots, s_{a_{ip}}, s_{b_{i0}}, \ldots, s_{b_{iq}} \right]^{\tau}$$

is the area coefficient vector, and

$$\boldsymbol{G}_i = \left[g_{a_{i1}}, \ldots, g_{a_{ip}}, g_{b_{i0}}, \ldots, g_{b_{iq}} \right]^{\tau}$$

is the gravity center coefficient vector.

Proof: According to Mamdani's implication, the m rules of Eq. (1) can be connected to form the following fuzzy relation

$$\boldsymbol{R} = \bigcup_{i=1}^{m} R_i = \bigcup_{i=1}^{m} (A_{i1} \times \ldots \times A_{ip} \times B_{i0} \times \ldots \times B_{iq} \times Y_i)$$

where Y_i indicates the fuzzy output Y of the ith rule. Then the fuzzy output set Y induced by the input vector $\boldsymbol{h}$ is given by

$$Y = \boldsymbol{h} \circ \boldsymbol{R} = \boldsymbol{h} \circ (\bigcup_{i=1}^{m} R_i).$$

For the convenience of analytic handling, substitute addition and product operators for $\wedge$ and $\vee$ operators (Mizumoto, 1991), and use the center of gravity defuzzification algorithm, the defuzzified output becomes

$$y(k) = \mathrm{Centroid}(Y) = \frac{\int_{-\infty}^{\infty} y \sum_{i=1}^{m} [\alpha_i \mu_{Y_i}(y)]\, dy}{\int_{-\infty}^{\infty} \sum_{i=1}^{m} [\alpha_i \mu_{Y_i}(y)]\, dy}. \qquad (4)$$

It follows that

$$y(k) = \frac{\sum_{i=1}^{m} \left\{ \frac{\int_{-\infty}^{\infty} y [\alpha_i \mu_{Y_i}(y)] dy \int_{-\infty}^{\infty} [\alpha_i \mu_{Y_i}(y)] dy}{\int_{-\infty}^{\infty} [\alpha_i \mu_{Y_i}(y)] dy} \right\}}{\sum_{i=1}^{m} \int_{-\infty}^{\infty} [\alpha_i \mu_{Y_i}(y)] dy}$$

$$= \frac{\sum_{i=1}^{m} \alpha_i (|\boldsymbol{h}|^{\tau} \boldsymbol{S}_i)(\boldsymbol{h}^{\tau} \boldsymbol{G}_i)}{\sum_{i=1}^{m} \alpha_i (|\boldsymbol{h}|^{\tau} \boldsymbol{S}_i)},$$

according to Eq. (4) and considering the following:

① The sum of two LR type fuzzy numbers is still an LR type fuzzy number, and the area of the sum equals the sum of the areas, the gravity center of the sum equals the sum of the gravity centers.

② The scalar product of an LR type fuzzy number and a real number is still an LR type fuzzy number, and the area of the product equals the product of the area of the fuzzy number and the absolute value of the real number, the gravity center of the product equals the product of the gravity center of the fuzzy number and the real number.

□

Corollary 1 If $s_{a_{ij}} = s_{b_{il}} = \text{Const.} > 0$, i=1,...,m, j=1,...,p, l=0,...,q then

$$y(k) = \text{Centroid}(Y) = \frac{\sum_{i=1}^{m} \alpha_i (\boldsymbol{h}^T \boldsymbol{G}_i)}{\sum_{i=1}^{m} \alpha_i}.$$

In this case, an ETSK model is equivalent to a TSK model. In other words, TSK model is a special case of ETSK model.

3. FUZZY IDENTIFICATION ALGORITHM BASED ON ETSK MODEL

Research in the identification of fuzzy models has yielded a wide variety of achievements (Kosko, 1991; Pedryce, 1984; Takagi, *et al.* 1985). In this section, an online fuzzy identification algorithm based on the ETSK model, described by Eq. (2), is proposed.

3.1 Identification of gravity center coefficient vector $\boldsymbol{G}_i$

For given input vector $\boldsymbol{h}$, α_i can be calculated from Eq. (3). If $\boldsymbol{S}_i$ is kept fixed, then Eq. (2) becomes

$$y(k) = \frac{\sum_{i=1}^{m} c_i (\boldsymbol{h}^T \boldsymbol{G}_i)}{c} = \boldsymbol{H}^T \boldsymbol{G} \qquad (5)$$

where

$$c = \sum_{i=1}^{m} c_i\ , \quad c_i = \alpha_i |\boldsymbol{h}|^T \boldsymbol{S}_i\ ,$$

$$\boldsymbol{H} = \left[\frac{c_1 \boldsymbol{h}^T}{c}, \frac{c_2 \boldsymbol{h}^T}{c}, \ldots, \frac{c_m \boldsymbol{h}^T}{c} \right]^T,$$

$$\boldsymbol{G} = \left[\boldsymbol{G}_1^T, \boldsymbol{G}_2^T, \ldots, \boldsymbol{G}_m^T \right]^T.$$

Eq. (5) is a least square format where $y(k)$ and $\boldsymbol{H}$ are measurable, so many identification methods, such as Least Square Method (LSM) or Forgetting Factor Method (FFM), can be applied to identify the parameter vector $\boldsymbol{G}_i$.

3.2 Identification of area coefficient vector $\boldsymbol{S}_i$

Given input vector $\boldsymbol{h}$ and $\boldsymbol{G}_i$ is kept fixed, then Eq. (2) becomes

$$y(k) = \frac{\sum_{i=1}^{m} d_i (|\boldsymbol{h}|^T \boldsymbol{S}_i)}{\sum_{i=1}^{m} \alpha_i (|\boldsymbol{h}|^T \boldsymbol{S}_i)} = \frac{\boldsymbol{H}_1^T \boldsymbol{S}}{\boldsymbol{H}_2^T \boldsymbol{S}} \qquad (6)$$

where

$$d_i = \alpha_i \boldsymbol{h}^T \boldsymbol{G}_i$$

$$\boldsymbol{S} = \left[\boldsymbol{S}_1^T, \boldsymbol{S}_2^T, \ldots, \boldsymbol{S}_m^T \right]^T,$$

$$\boldsymbol{H}_1 = \left[d_1 |\boldsymbol{h}|^T, d_2 |\boldsymbol{h}|^T, \ldots, d_m |\boldsymbol{h}|^T \right]^T,$$

$$\boldsymbol{H}_2 = \left[\alpha_1 |\boldsymbol{h}|^T, \alpha_2 |\boldsymbol{h}|^T, \ldots, \alpha_m |\boldsymbol{h}|^T \right]^T.$$

Clearly, $y(k)$, $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ in Eq. (6) can be measured, so instantaneous training algorithms, such as Least Mean Square (LMS) or Normalized Least Mean Square (NLMS), can be adopted to identify the parameter vector $\boldsymbol{S}_i$ in Eq. (6).

3.3 Identification algorithm

The online identification algorithm of an ETSK Model is described as follows

I Choose the original values of $\boldsymbol{h}$, $\boldsymbol{G}$, $\boldsymbol{S}$ and cycle index N;

II For step k

① Keep $\boldsymbol{S}_i$ fixed and utilize Recursive Forgetting Factor (RFF) method to identify the gravity center coefficient vector $\boldsymbol{G}_i$;

② Keep $\boldsymbol{G}_i$ fixed and utilize Normalized Least Mean Square (NLMS) algorithm to identify the area coefficient vector $\boldsymbol{S}_i$;

III k=k+1; If k<N then go back to II, else end identification.

4. VARIABLE WEIGHTS TSK MODEL

The LR type fuzzy numbers and extended operations in the consequent of the rules increase the interpreting ability of ETSK model, but they also increase the difficulty in the fuzzy controller design in which extended real operations, especially extended division, are inevitable. In this section, a kind of Variable Weights TSK (VWTSK) model that is equivalent to ETSK model is induced. So that fuzzy numbers and extended operations in the consequent of the rules of an ETSK model are converted to real numbers and ordinary operations.

A type of VWTSK model is composed of rules in the form

IF $y(k\text{-}1)$ is A_{i1} AND $y(k\text{-}2)$ is A_{i2}
AND,...,AND $y(k\text{-}p)$ is A_{ip}
AND $u(k)$ is B_{i0} AND $u(k\text{-}1)$ is B_{i1}
AND,...,AND $u(k\text{-}q)$ is B_{iq}
THEN $y(k) = g_{a_{i1}} y(k\text{-}1) +,\ldots,+ g_{a_{ip}} y(k\text{-}p)$
$+ g_{b_{i0}} u(k) +,\ldots,+ g_{b_{iq}} u(k\text{-}q)$, WITH $\omega_i(\boldsymbol{h})$ (7)

where $u(k)$ and $y(k)$ are input and output of the model at k time; $y(k\text{-}j)$, $u(k\text{-}t)$, $g_{a_{ij}}, g_{b_{it}} \in \mathrm{R}^1$; A_{ij} and B_{it} are fuzzy input sets of the corresponding universe of discourse; $\omega_i(\boldsymbol{h})$ is the weight of the ith rule, $\boldsymbol{h}$ is the input vector denoted in Theorem 1; i=1,...,m, j=1,...,p, t=0,...,q.

Theorem 2 Given a fuzzy system F: $\mathrm{R}^{p+q+1} \to \mathrm{R}^1$ described by Eq. (7), if

$$\omega_i(\boldsymbol{h}) = \frac{|\boldsymbol{h}|^T \boldsymbol{S}_i}{\sum_{j=1}^{m}\left[|\boldsymbol{h}|^T \boldsymbol{S}_j\right]} \tag{8}$$

then it is equivalent to the fuzzy system described by Eq. (1) that is they have the same input-output relation

$$y(k) = \frac{\sum_{i=1}^{m}\alpha_i(|\boldsymbol{h}|^T \boldsymbol{S}_i)(\boldsymbol{h}^T \boldsymbol{G}_i)}{\sum_{i=1}^{m}\alpha_i(|\boldsymbol{h}|^T \boldsymbol{S}_i)}.$$

Proof: For the VWTSK model described by Eq. (7) it follows from Eq. (8) that

$$y(k) = \frac{\int_{-\infty}^{\infty} y \sum_{i=1}^{m}\left[\alpha_i \omega_i(\boldsymbol{h})\delta(y - y_i(k))\right]dy}{\int_{-\infty}^{\infty}\sum_{i=1}^{m}\left[\alpha_i \omega_i(\boldsymbol{h})\delta(y - y_i(k))\right]dy}$$

$$= \frac{\sum_{i=1}^{m}\left[\alpha_i \omega_i(\boldsymbol{h})\int_{-\infty}^{\infty} y\delta(y - y_i(k))dy\right]}{\sum_{i=1}^{m}\left[\alpha_i \omega_i(\boldsymbol{h})\int_{-\infty}^{\infty} \delta(y - y_i(k))dy\right]}$$

$$= \frac{\sum_{i=1}^{m}\alpha_i\omega_i(\boldsymbol{h})y_i(k)}{\sum_{i=1}^{m}\alpha_i\omega_i(\boldsymbol{h})} = \frac{\sum_{i=1}^{m}\alpha_i\omega_i(\boldsymbol{h})(\boldsymbol{h}^T\boldsymbol{G}_i)}{\sum_{i=1}^{m}\alpha_i\omega_i(\boldsymbol{h})}$$

$$= \frac{\sum_{i=1}^{m}\alpha_i \dfrac{|\boldsymbol{h}|^T \boldsymbol{S}_i}{\sum_{j=1}^{m}|\boldsymbol{h}|^T \boldsymbol{S}_j}(\boldsymbol{h}^T\boldsymbol{G}_i)}{\sum_{i=1}^{m}\alpha_i \dfrac{|\boldsymbol{h}|^T \boldsymbol{S}_i}{\sum_{j=1}^{m}|\boldsymbol{h}|^T \boldsymbol{S}_j}}$$

$$= \frac{\sum_{i=1}^{m}\alpha_i|\boldsymbol{h}|^T \boldsymbol{S}_i(\boldsymbol{h}^T\boldsymbol{G}_i)}{\sum_{i=1}^{m}\alpha_i|\boldsymbol{h}|^T \boldsymbol{S}_i}.$$

Thus, Theorem 2 has been proved. □

Theorem 2 provides a direct relationship between the gravity center coefficient vector $\boldsymbol{G}_i$, the area coefficient vector $\boldsymbol{S}_i$, the input vector $\boldsymbol{h}$ and weights. ETSK model is equivalent to a kind of VWTSK model in which the weights of the rules are determined by system inputs and area coefficient vectors, and the linear parameters of the consequent are gravity center coefficient vectors.

5. FUZZY CONTROL ALGORITHM BASED ON AN ETSK MODEL

Theorem 2 offers us a method to construct a fuzzy control algorithm from an ETSK model. It can be expressed as:

I Transform the ETSK model to the VWTSK model by using Theorem 2;

II Design the corresponding control algorithm $u(k+1) = R_i(z^{-1})$ in accordance with the consequent of the ith rule of the VWTSK Model

$$y(k) = g_{a_{i1}}y(k\text{-}1) + \dots + g_{a_{ip}}y(k\text{-}p)$$
$$+ g_{b_{i0}}u(k) + \dots + g_{b_{iq}}u(k\text{-}q).$$

For example, $R_i(z^{-1})$ can be Deadbeat controller (DB) or Minimum Variance controller (MV);

III Keep the antecedent of the ith rule of the VWTSK Model fixed, and substitute $\mathrm{R}_i(z^{-1})$ for its consequent;

IV $w_i(\boldsymbol{h})$, the weight of the ith rule of the fuzzy control algorithm, equals to $\omega_i(\boldsymbol{h})$, that is

$$w_i(\boldsymbol{h}) = \frac{|\boldsymbol{h}|^T \boldsymbol{S}_i}{\sum_{j=1}^{m}\left[|\boldsymbol{h}|^T \boldsymbol{S}_j\right]}.$$

From the step above (I to IV), the fuzzy control algorithm based on an ETSK model is composed of rules in the form

IF $y(k\text{-}1)$ is A_{i1} AND $y(k\text{-}2)$ is A_{i2} AND,...,AND $y(k\text{-}p)$ is A_{ip} AND $u(k)$ is B_{i0} AND $u(k\text{-}1)$ is B_{i1} AND,...,AND $u(k\text{-}q)$ is B_{iq}

THEN $u(k+1) = R_i(z^{-1})e(k)$, WITH $w_i(\boldsymbol{h})$

where $e(k)=r(k)-y(k)$; $r(k)$ is the command input; $y(k\text{-}j)$, $u(k\text{-}t)$, $e(k)$, $r(k) \in \mathrm{R}^1$; A_{ij} and B_{it} are fuzzy input sets of the corresponding universe of discourse; $\boldsymbol{h}$ is the input vector; $w_i(\boldsymbol{h})$ is the weight of the ith rule; j=1,...,p, t=0,...,q.

6. SIMULATION

6.1 Identification of ETSK model

Consider a nonlinear system

$$y(k) = 0.3y(k\text{-}1)u(k\text{-}2) - 0.7y(k\text{-}2) + 0.4u(k\text{-}1)$$
$$\cos(0.7y(k\text{-}1)) + 0.3u^3(k\text{-}1) + 0.3y^3(k\text{-}1) \tag{9}$$

where $u(k)$ is a uniformly distributed random signal on [-1,1]; $y(0)=y(1)=u(0)=u(1)=0$.

The fuzzy partition of $y(k\text{-}1)$, $y(k\text{-}2)$, $u(k\text{-}1)$ and $u(k\text{-}2)$ are shown in Fig. 1.

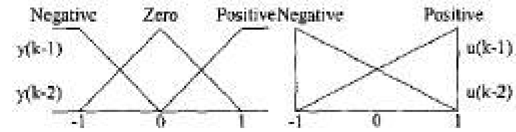

Fig.1 Fuzzy partition of $y(k\text{-}1)$, $y(k\text{-}2)$, $u(k\text{-}1)$ and $u(k\text{-}2)$

Using the method proposed by Takagi (Takagi, *et al*, 1985), a Linguistic model comprised of 36 rules is obtained after 300 iterations:

IF $y(k\text{-}1)$ is Negative AND $y(k\text{-}2)$ is Negative AND $u(k\text{-}1)$ is Negative AND $u(k\text{-}2)$ is Negative THEN $y(k)= 0.5109$ ALSO

IF $y(k\text{-}1)$ is Negative AND $y(k\text{-}2)$ is Negative AND $u(k\text{-}1)$ is Negative AND $u(k\text{-}2)$ is Positive THEN $y(k)=-0.8717$ ALSO

......

IF $y(k\text{-}1)$ is Positive AND $y(k\text{-}2)$ is Positive AND $u(k\text{-}1)$ is Positive AND $u(k\text{-}2)$ is Positive THEN $y(k)= 0.1102$ (10)

Using the same method and iterations, a TSK model comprised of 3 rules is obtained:

IF $y(k\text{-}1)$ is Negative
THEN $y(k)=0.2413y(k\text{-}1)\text{-}0.6724y(k\text{-}2)+0.5107\,u(k\text{-}1)\text{-}0.2376u(k\text{-}2)$ ALSO
IF $y(k\text{-}1)$ is Zero
THEN $y(k)=-0.1279y(k\text{-}1)\text{-}0.7159y(k\text{-}2)+0.6112\,u(k\text{-}1)\text{-}0.0294u(k\text{-}2)$ ALSO
IF $y(k\text{-}1)$ is Positive
THEN $y(k)=0.3263y(k\text{-}1)\text{-}0.6679y(k\text{-}2)+0.4439u(k\text{-}1)+0.3145u(k\text{-}2)$ (11)

Using the identification algorithm in section 3 and after 300 iterations, an ETSK model comprised of 3 rules is obtained:

IF $y(k\text{-}1)$ is Negative
THEN $y(k)= (1.2196,0.2505)\otimes y(k\text{ -}1)\oplus(1.0968,\text{-}.6707)\otimes y(k\text{-}2)\oplus(0.9246,0.5069)\otimes u(k\text{-}1)\oplus(1.1780,\text{-}0.2545)\otimes u(k\text{ -}2)$ ALSO
IF $y(k\text{ -}1)$ is Zero
THEN $y(k)=(1.3220,\text{-}0.1061)\otimes y(k\text{-}1)\oplus(1.1091,\text{-}0.7145)\otimes y(k\text{-}2)\oplus(1.2341,0.6076)\otimes u(k\text{-}1)\oplus(1.1132,\text{-}0.0272)\otimes u(k\text{-}2)$ ALSO
IF $y(k\text{-}1)$ is Positive
THEN $y(k)= (0.7047,0.3221)\otimes y(k\text{-}1)\oplus(1.0465,\text{-}0.6701)\otimes y(k\text{-}2)\oplus(1.0841,0.4369)\otimes u(k\text{-}1)\oplus(1.0743,0.3240)\otimes u(k\text{-}2)$ (12)

where (*,*) denotes an LR type fuzzy number's area and gravity center.

Fig. 2 shows the five-step prediction error of the LM model (10), the TSK model (11) and the ETSK model (12). It can be seen that the prediction error of the ETSK model has the least mean $E\{e(k)\}$ and variance $Var\{e(k)\}$. The other experiments also show that for different prediction steps, the ETSK model always outperforms the LM model and the TSK model under the same conditions.

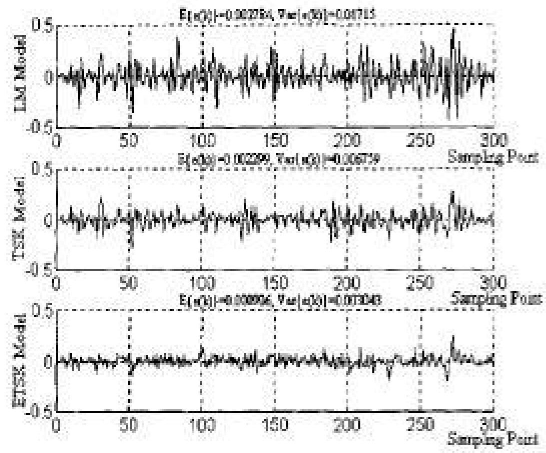

Fig 2. Five step prediction error of LM (10), TSK model (11) and ETSK model (12)

6.2 Example of control algorithm based on ETSK model

Consider the nonlinear system (9), a fuzzy PID control algorithm and a control algorithm based on the ETSK model are designed to trace square wave signal $r(k)$.

The fuzzy PID control algorithm comprised of 18 rules is designed as follows

IF $e(k)$ is Negative AND $\sum e(k)$ is Negative AND $\Delta e(k)$ is Negative
THEN $u(k)= -0.9$ ALSO
IF $e(k)$ is Negative AND $\sum e(k)$ is Zero AND $\Delta e(k)$ is Negative
THEN $u(k)= -0.3$ ALSO
IF $e(k)$ is Negative AND $\sum e(k)$ is Positive AND $\Delta e(k)$ is Negative
THEN $u(k)= 0.0$ ALSO

......

IF $e(k)$ is Positive AND $\sum e(k)$ is Positive AND $\Delta e(k)$ is Positive
THEN $u(k)=0.9$ (13)

The control algorithm based on the ETSK model is designed as follows

IF $y(k\text{-}1)$ is Negative
THEN $u(k)=0.0203u(k\text{-}1)\text{-}0.0596e(k)\text{-}0.3896e(k\text{-}1)+1.7087r(k)$,
WITH $w_1=(1.2196|y(k\text{-}1)|+1.0968|y(k\text{-}2)|+0.9246|u(k\text{-}1)|+1.1780|u(k\text{-}2)|)/s(k)$ ALSO
IF $y(k\text{-}1)$ is Zero
THEN $u(k)=0.0304u(k\text{-}1)+0.4092e(k)\text{-}1.2518e(k\text{-}1)+2.4418r(k)$,
WITH $w_2=(1.3220|y(k\text{-}1)|+1.1091|y(k\text{-}2)|+1.2341|u(k\text{-}1)|+1.1132|u(k\text{-}2)|)/s(k)$ ALSO
IF $y(k\text{-}1)$ is Positive
THEN $u(k)=0.0120u(k\text{-}1)+0.3004e(k)\text{-}0.5572e(k\text{-}1)+1.2286r(k)$,

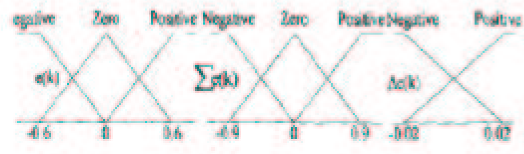

Fig.3. Fuzzy partition of $e(k)$, $\Delta e(k)$ and $\sum e(k)$

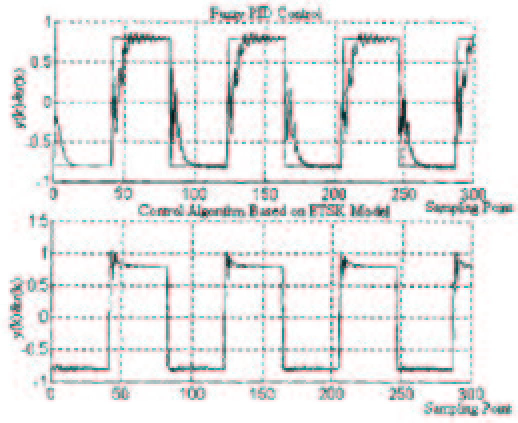

Fig. 4. Trace a square wave signal

$$\text{WITH } w_1=(0.7047|y(k-1)|+1.0465|y(k-2)| +1.0841|u(k-1)|+1.0743|u(k-2)|)/s(k) \tag{14}$$

where $s(k)$ is defined as

$$s(k)=3.2463|y(k-1)|+3.2524|y(k-2)| +3.2428|u(k-1)|+3.3655|u(k-2)|$$

In Eqs. (13) and (14), $e(k)$ is the trace error which is equal to $r(k)$-$y(k)$, the error change $\Delta e(k)=e(k)-e(k-1)$, and $\sum e(k)$ is the error cumulation. They are taken as the inputs of the fuzzy PID controller. $u(k)$ is the output of the controllers. The fuzzy partition of $y(k-1)$ is shown in Fig.1 and the fuzzy partition of $e(k)$, $\Delta e(k)$ and $\sum e(k)$ are shown in Fig.3.

The control results are shown in Fig. 4. The performance of the controller based on ETSK model is superior to the fuzzy PID controller.

7. CONCLUSIONS

First, an ETSK model can give out more accurate long-range predications than a Linguistic model and a TSK model can do, and the number of its rules is less than that of a Linguistic Model. The identification algorithm proposed in section 3 is very effective and simple.

Second, if an ETSK model has been acquired, then comply with the steps I-IV in section 5, a control algorithm based on the ETSK model can be designed. The consequent of its rules — $R_i(z^{-1})$ is selected in accordance with the specific cases. Comparing with fuzzy PID control, this control algorithm can achieve better control performance.

Finally, the combination of an ETSK model, identification algorithm and control algorithm will produce a new strategy of fuzzy adaptive control.

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