Multiparadigm Programming with Python 3
Chapter 5

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**Browser Advisory:** The HTML version of this textbook requires a browser that supports the display of MathML. A good choice as of September 2018 is a recent version of Firefox from Mozilla.
5 Python 3 Types

5.1 Chapter Introduction

The goals of this chapter are to:

- examine the general concepts of type systems
- explore Python 3’s type system and built-in types

5.2 Type System Concepts

The term type tends to be used in many different ways in programming languages. What is a type?

5.2.1 Types and subtypes

Conceptually, a type is a set of values (i.e. possible states or objects) and a set of operations defined on the values in that set.

Similarly, a type S is (a behavioral) subtype of type T if the set of values of type S is a “subset” of the values in set T and set of operations of type S is a “superset” of the operations of type T. That is, we can safely substitute elements of subtype S for elements of type T because S’s operations behave the “same” as T’s operations.

This is known as the Liskov Substitution Principle [Liskov 1987] [Wikipedia 2018a].

Consider a type representing all furniture and a type representing all chairs. In general, we consider the set of chairs to be a subset of the set of furniture. A chair should have all the general characteristics of furniture, but it may have additional characteristics specific to chairs.

If we can perform an operation on furniture in general, we should be able to perform the same operation on a chair under the same circumstances and get the same result. Of course, there may be additional operations we can perform on chairs that are not applicable to furniture in general.

Thus the type of all chairs is a subtype of the type of all furniture according to the Liskov Substitution Principle.

5.2.2 Constants, variables, and expressions

Now consider the types of the basic program elements.
A constant has whatever types it is defined to have in the context in which it is used. For example, the constant symbol 1 might represent an integer, a real number, a complex number, a single bit, etc., depending upon the context.

A variable has whatever types its value has in a particular context and at a particular time during execution.

An expression has whatever types its evaluation yields based on the types of the variables, constants, and operations from which it is constructed.

5.2.3 Static and dynamic

In a statically typed language, the types of a variable or expression can be determined from the program source code and checked at “compile time” (i.e. during the syntactic and semantic processing in the front-end of a language processor). Such languages may require at least some of the types of variables or expressions to be declared explicitly, while others may be inferred implicitly from the context.

Java, Scala, and Haskell are examples of statically typed languages.

In a dynamically typed language, the specific types of a variable or expression cannot be determined at “compile time” but can be checked at runtime.

Lisp, Python, JavaScript, and Lua are examples of dynamically typed languages.

Of course, most languages use a mixture of static and dynamic typing. For example, Java objects defined within an inheritance hierarchy must be bound dynamically to the appropriate operations at runtime. Also Java objects declared of type Object (the root class of all user-defined classes) often require explicit runtime checks or coercions.

5.2.4 Nominal and structural

In a language with nominal typing, the type of value is based on the type name assigned when the value is created. Two values have the same type if they have the same type name. A type $S$ is a subtype of type $T$ only if $S$ is explicitly declared to be a subtype of $T$.

For example, Java is primarily a nominally typed language. It assigns types to an object based on the name of the class from which the object is instantiated and the superclasses extended and interfaces implemented by that class.

However, Java does not guarantee that subtypes satisfy the Liskov Substitution Principle. For example, a subclass might not implement an operation in a manner that is compatible with the superclass. (The behavior of subclass objects are this different from the behavior of superclass objects.) Ensuring that Java subclasses preserve the Substitution Principle is considered good programming practice in most circumstances.
In a language with *structural typing*, the type of a value is based on the *structure* of the value. Two values have the same type if they have the “same” structure; that is, they have the same *public* data attributes and operations and these are themselves of compatible types.

In structurally typed languages, a type $S$ is a subtype of type $T$ only if $S$ has all the public data values and operations of type $T$ and the data values and operations are themselves of compatible types. Subtype $S$ may have additional data values and operations not in $T$.

Haskell is an example of a primarily structurally typed language.

### 5.2.5 Polymorphic operations

*Polymorphism* refers to the property of having “many shapes”. In programming languages, we are primarily interested in how *polymorphic* function names (or operator symbols) are associated with implementations of the functions (or operations).

In general, two primary kinds of polymorphic operations exist in programming languages:

1. *Ad hoc polymorphism*, in which the same function name (or operator symbol) can denote different implementations depending upon how it is used in an expression. That is, the implementation invoked depends upon the types of function’s arguments and return value.

   There are two subkinds of ad hoc polymorphism.

   a. *Overloading* refers to ad hoc polymorphism in which the language’s compiler or interpreter determines the appropriate implementation to invoke using information from the context. In statically typed languages, overloaded names and symbols can usually be bound to the intended implementation at *compile time* based on the declared types of the entities. They exhibit *early binding*.

      Consider the language Java. It overloads a few operator symbols, such as using the $+$ symbol for both addition of numbers and concatenation of strings. Java also overloads calls of functions defined with the same name but different signatures (patterns of parameter types and return value). Java does not support user-defined operator overloading; C++ does.

      Haskell’s *type class* mechanism implements overloading polymorphism in Haskell. There are similar mechanisms in other languages such as Scala and Rust.

   b. *Subtyping* (also known as *subtype polymorphism* or *inclusion polymorphism*) refers to ad hoc polymorphism in which the appropriate
implementation is determined by searching a hierarchy of types. The function may be defined in a supertype and redefined (overridden) in subtypes. Beginning with the actual types of the data involved, the program searches up the type hierarchy to find the appropriate implementation to invoke. This usually occurs at runtime, so this exhibits late binding.

The object-oriented programming community often refers to inheritance-based subtype polymorphism as simply polymorphism. This is the polymorphism associated with the class structure in Java.

Haskell does not support subtyping. Its type classes do support class extension, which enables one class to inherit the properties of another. However, Haskell’s classes are not types.

2. **Parametric polymorphism**, in which the same implementation can be used for many different types. In most cases, the function (or class) implementation is stated in terms of one or more type parameters. In statically typed languages, this binding can usually be done at compile time (i.e. exhibiting early binding).

The object-oriented programming (e.g. Java) community often calls this type of polymorphism generics or generic programming.

The functional programming (e.g. Haskell) community often calls this simply polymorphism.

Todo: Bring “row polymorphism” into the above discussion?

### 5.2.6 Polymorphic variables

A **polymorphic variable** is a variable that can “hold” values of different types during program execution.

For example, a variable in a dynamically typed language (e.g. Python) is polymorphic. It can potentially “hold” any value. The variable takes on the type of whatever value it “holds” at a particular point during execution.

Also, a variable in a nominally and statically typed, object-oriented language (e.g. Java) is polymorphic. It can “hold” a value its declared type or of any of the subtypes of that type. The variable is declared with a static type; its value has a dynamic type.

A variable that is a parameter of a (parametrically) polymorphic function is polymorphic. It may be bound to different types on different calls of the function.

### 5.3 Python 3 Type System

What about Python 3’s type system?
5.3.1 Objects

Python 3 is object-based [Cunningham 2018b, Ch. 3]; it treats all data as objects. A Python 3 object has the following essential characteristics of objects [Cunningham 2018b, Ch. 3]:

a. a state (value) drawn from a set of possible values
   The state may consist of several distinct data attributes. In this case, the set of possible values is the Cartesian product of the sets of possible values of each attribute.

b. a set of operations that access and/or mutate the state

c. a unique identity (e.g. address in memory)

A Python 3 object has one of the two important but nonessential characteristics of objects [Cunningham 2018b, Ch. 3]. Python 3 does:

d. not enforce encapsulation of the state within the object, instead relying upon programming conventions and name obfuscation to hide private information

e. exhibit an independent lifecycle (i.e. has a different lifetime than the code that created it)

As we see later chapter, each object has a distinct dictionary, the directory, that maps the local names to the data attributes and operations.

Python 3 typically uses dot notation to access an object’s data attributes and operations:

- `obj.data` accesses the attribute `data` of `obj`
- `obj.op` accesses operation `op` of `obj`
- `obj.op()` invokes operation `op` of `obj`, passing any arguments in a comma-separated list between the parentheses

Some objects are immutable and others are mutable. The states (i.e. values) of:

- immutable objects (e.g. numbers, booleans, strings, and tuples) cannot be changed after creation
- mutable objects (e.g. lists, dictionaries, and sets) can be changed in place after creation

Caveat: We cannot modify a Python 3 tuple’s structure (i.e., length) after its creation. However, if the components of a tuple are themselves mutable objects, they can be changed in-place.

All Python 3 objects have a type.
5.3.2 Types

In terms of the discussion in Type System Concepts section, all Python 3 objects can be considered as having one or more conceptual types at a particular point in time. The types may change over time because the program can change the possible set of data attributes and operations associated with the object.

A Python 3 variable is bound to an object by an assignment statement or its equivalent. Python 3 variables are thus dynamically typed, as are Python expressions.

Although a Python 3 program usually constructs an object within a particular nominal type hierarchy (e.g. as an instance of a class), this may not fully describe the type of the object, even initially. And the ability to dynamically add, remove, and modify attributes (both data fields and operations) means the type can change as the program executes.

The type of a Python 3 object is determined by what it can do—what data it can hold and what operations it can perform on that data—rather than how it was created. We sometimes call this dynamic, structural typing approach duck typing. (If it walks like a duck and quacks like a duck, then it is a duck, even if is declared as a snake.)

For example, we can say that any object is of an iterable type if it implements an __iter__ operation that returns a valid iterator object. An iterator object must implement a __next__ operation that retrieves the next element of the “collection” and must raise a StopIteration exception if no more elements are available.

In Python 3, we sometimes refer to a type like iterable as a protocol. That is, it is a, perhaps informal, interface that objects are expected to satisfy in certain circumstances.

5.4 Built-in Types

Python 3 provides several built-in types and subtypes, which are named and implemented in the core language. When displayed, these types are shown as follows:

```
<class 'int'>
```

That is, the value is an instance of a class named int. Python 3 uses the term class to describe its nominal types.

We can query the nominal type of an object obj with the function call type(obj). In the following discussion, we show the results from calling this function interactively in Python 3 REPL (Read-Evaluate-Print Loop) sessions.

For the purpose of our discussion, the primary built-in types include:
5.4.1 Singleton types

Python 3 has single-element types used for special purposes. These include None and NotImplemented.

5.4.1.1 None

The name None denotes a value of a singleton type. That is, the type has one element written as None.

Python 3 programs normally use None to mean there is no meaningful value of another type. For example, this is the value returned by Python 3 procedures.

5.4.1.2 NotImplemented

The name NotImplemented also denotes a value of a singleton type. Python programs normally use this value to mean that an arithmetic or comparison operation is not implemented on the operand objects.

5.4.2 Number types

Core Python 3 supports four types of numbers:

- integers
- real numbers
- complex numbers
- Booleans

5.4.2.1 Integers (int)

Type int denotes the set of integers. They are encoded in a variant of two’s complement binary numbers in the underlying hardware. They are of unbounded precision, but they are, of course, limited in size by the available virtual memory.

```
>>> type(1)
<class 'int'>
>>> type(-14)
```
5.4.2.2 Real numbers (float)

Type float denotes the subset of the real numbers that can be encoded as double precision floating point numbers in the underlying hardware.

```python
>>> type(1.01)
<class 'float'>
>>> type(-14.3)
<class 'float'>
>>> x = 2
>>> type(x)
<class 'int'>
>>> y = 2.0
>>> type(y)
<class 'float'>
>>> x == y  # Note result of equality comparison
True
```

5.4.2.3 Complex numbers (complex)

Type complex denotes a subset of the complex numbers encoded as a pair of floats, one for the real part and one for the imaginary part.

```python
>>> type(complex('1+2j'))  # real part 1, imaginary part 2
<class 'complex'>
>>> complex('1') == 1.0  # Note result of comparison
True
>>> complex('1') == 1    # Note result of comparison
True
```

5.4.2.4 Booleans (bool)

Type bool denotes the set of Boolean values False and True. In Python, this is a subtype of int with False and True having the values 0 and 1, respectively.

```python
>>> type(False)
<class 'bool'>
>>> type(True)
<class 'bool'>
```
>>> True == 1
True

Making bool a subtype of int is an unfortunate legacy design choice from the early days of Python. It is better not to rely on this feature in Python 3 programs.

5.4.2.5 Truthy and falsy values

Python 3 programs can test any object as if it was a Boolean (e.g. within the condition of an if or while statement or as an operand of a Boolean operation).

An object is falsy (i.e. considered as False) if its class defines

- a special method __bool__() that, when called on the object, returns False
- a special method __len__() that returns 0

Note: We discuss special methods in a later chapter.

Otherwise, the object is truthy (i.e. considered as True).

The singleton value NotImplemented is explicitly defined as truthy.

Falsy built-in values include:

- constants False and None
- numeric values of zero such as 0, 0.0, and 0j
- empty sequences and collections such as '', [], [], and {} (defined below)

Unless otherwise documented, any function expected to return a Boolean result should return False or 0 for false and True or 1 for true. However, the Boolean operations or and and should always return one of their operands.

5.4.3 Sequence types

A sequence denotes a serially ordered collection of zero or more objects. An object may occur more than once in a sequence.

Python 3 supports a number of core sequence types. Some sequences have immutable structures and some have mutable.

5.4.3.1 Immutable sequences

An immutable sequence is a sequence in which the structure cannot be changed after initialization.
5.4.3.1  str

Type str (string) denotes sequences of text characters—that is, of Unicode code points in Python 3. We can express strings syntactically by putting the characters between single, double, or triple quotes. The latter supports multi-line strings.

Python does not have a separate character type; a charactor is a single-element str.

```python
>>> type('Hello world')
<class 'str'>
>>> type("Hi Earth")
<class 'str'>
>>> type('... Can have embedded newlines ...
... ')
<class 'str'>
```

5.4.3.1.2  tuple

Type tuple type denotes fixed length, heterogeneous sequences of objects. We can express tuples syntactically as sequences of comma-separated expressions in parentheses.

The tuple itself is immutable, but the objects in the sequence might themselves be mutable.

```python
>>> type(()  
<class 'tuple'>
>>> type((1,))  # one-element tuple, note comma
<class 'tuple'>
>>> x = (1,'Ole Miss')  # mixed element types
>>> type(x)
<class 'tuple'>
>>> x[0]  # access element with index 0
1
>>> x[1]  # access element with index 1
'Ole Miss'
```

5.4.3.1.3  range

The range type denotes an immutable sequence of numbers. It is commonly used to specify loop controls.

- **range(stop)** denotes the sequence of integers that starts from 0, increases by steps of 1, and stops at stop-1; if stop <= 0, the range is empty.
• **range(start, stop)** denotes the sequence of integers that starts from
  **start**, increases by steps of 1, and stops at **stop-1**; if **stop <= start**, the range is empty.

• **range(start, stop, step)** denotes the sequence of integers that starts from **start** with a **nonzero** steps of **step**.

  If **step** is positive, the sequence increases toward **stop-1**; if **stop <= start**, the range is empty.

  if negative, the sequence decreases toward **stop+1**.

  A range is a lazy data structure. It only yields a value if the output is needed.

```python
>>> list(range(5))
[0, 1, 2, 3, 4]
>>> list(range(1, 5))
[1, 2, 3, 4]
>>> list(range(0, 9, 3))
[0, 3, 6]
>>> list(range(0, 10, 3))
[0, 3, 6, 9]
>>> list(range(5, 0, -1))
[5, 4, 3, 2, 1]
>>> list(range(0))
[]
>>> list(range(1, 0))
[]
>>> list(range(0, 0))
[]
```

5.4.3.1.4 **bytes**

Type **bytes** type denotes sequences of 8-bit bytes. We can express these syntactically as **ASCII character strings** prefixed by a “b”.

```python
>>> type(b'Hello\n World!')
<class 'bytes'>
```

5.4.3.2 **Mutable sequences**

A mutable sequence is a sequence in which the structure can be changed after initialization.

5.4.3.2.1 **list**
Type `list` denotes variable-length, heterogeneous sequences of objects. We can express lists syntactically as comma-separated sequence of expressions between square brackets.

```python
>>> type([])
<class 'list'>
>>> type([3])
<class 'list'>
>>> x = [1, 2, 3] + ['four', 'five']  # concatenation
>>> x
[1, 2, 3, 'four', 'five']
>>> type(x)
<class 'list'>
>>> y = x[1:3]  # get slice of list
>>> y
[2, 3]
>>> y[0] = 3  # assign to list index 0
[3, 3]
```

5.4.3.2.2 `bytearray`

Type `bytearray` denotes mutable sequences of 8-bit bytes, that is otherwise like type `bytes`. They are constructed by calling the function `bytearray()`.

```python
>>> type(bytearray(b'Hello
 World!'))
<class 'bytes'>
```

5.4.4 Mapping types

Type `dict` (dictionary) denotes mutable finite sets of key-value pairs, where the key is an index into the set for the value with which it is paired.

The key can be any `hashable` object. That is, the key can be any immutable object or an object that always gives the same hash value. However, the associated value objects may be mutable and the membership in the set may change.

We can express dictionaries syntactically in various ways such as comma-separated lists of key-value pairs with braces.

```python
>>> x = { 1 : "one" }
>>> x
{1: 'one'}
>>> type(x)
<class 'dict'>
>>> x[1]
'one'
>>> x.update({ 2 : "two" })  # add to dictionary
```
>>> x
{1: 'one', 2: 'two'}
>>> type(x)
<class 'dict'>
>>> del x[1]  # delete element with key
>>> x
{2: 'two'}

5.4.5 Set Types

A set is an unordered collection of distinct hashable objects.

There are two built-in set types—set and frozenset.

5.4.5.1 set

Type set denotes a mutable collection.

We can create a nonempty set by putting a comma-separated list of elements between braces as well as by using the set constructor.

For example, sets sx and sy below have the same elements. The operation |= adds the elements of the right operand to the left.

```python
>>> sx = { 'Dijkstra', 'Hoare', 'Knuth' }
>>> sx
{'Knuth', 'Hoare', 'Dijkstra'}
>>> sy = set(['Knuth', 'Dijkstra', 'Hoare'])
>>> sy
{'Knuth', 'Hoare', 'Dijkstra'}
>>> sx == sy
True
>>> sx.add('Turing')  # add element to mutable set
>>> sx
{'Turing', 'Knuth', 'Hoare', 'Dijkstra'}
```

5.4.5.2 frozenset

Type frozenset denotes an immutable collection.

We can extend the set example above as follows:

```python
>>> fx = frozenset(['Dijkstra', 'Hoare', 'Knuth'])
>>> fx
frozenset({'Knuth', 'Hoare', 'Dijkstra'})
>>> fy = frozenset(sy)
>>> fy
frozenset({'Knuth', 'Hoare', 'Dijkstra'})
```
>>> fx.add('Turing')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
AttributeError: 'frozenset' object has no attribute 'add'

5.4.6 Other object types

We discuss callable objects (e.g. functions), class objects, module objects, and user-defined types (classes) in later chapters.

5.5 What Next?

TODO

5.6 Exercises

TODO

5.7 Acknowledgements

In Spring 2018, I wrote the general Type System Concepts section as a part of a chapter that discusses the type system of Python 3 [Cunningham 2018a].

In Summer 2018, I revised it to become Section 5.2 in the new Chapter 5 of the textbook Exploring Languages with Interpreters and Functional Programming (ELIFP) [Cunningham 2018b]. I also moved the “Kinds of Polymorphism” discussion from the 2017 List Programming chapter of that book to the new subsection “Polymorphic Operations”. (This section draws on various Wikipedia articles [Wikipedia 2018b] and other sources.)

In Fall 2018, I copied the general concepts from ELIFP and recombined it with the Python-specific content from the first part of [Cunningham 2018a] to form this chapter.

This chapter seeks to be compatible with the concepts, terminology, and approach of the 2018 version of my textbook Exploring Languages with Interpreters and Functional Programming [Cunningham 2018b], in particular of Chapters 2, 3, and 5.

I maintain this chapter as text in Pandoc’s dialect of Markdown using embedded LaTeX markup for the mathematical formulas and then translate the document to HTML, PDF, and other forms as needed.
5.8 References


5.9 Terms and Concepts

Object, object characteristics (state, operations, identity, encapsulation, independent lifecycle), immutable vs. mutable, type, subtype, Liskov Substitution Principle, types of constants, variables, and expressions, static vs. dynamic types, declared and inferred types, nominal vs. structural types, polymorphic operations (ad hoc, overloading, subtyping, parametric/generic), early vs. late binding, compile time vs. runtime, polymorphic variables, duck typing, protocol, interface, REPL, singleton types (*None* and *NotImplemented*), number types (*int, float, complex, bool, False, falsy, True, truthy), immutable sequence types (*str, tuple, range, bytes*), mutable sequence types (*list, bytearray*), mapping types (*dict, key and value*), set types (*set, frozenset*), other types.