# CSci 555: Functional Programming Functional Programming in Scala Strictness and Laziness

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**Note:** This set of notes accompanies my lectures on Chapter 5 of the book Functional Programming in Scala [Chiusano 2015] (i.e. the Red Book).

**Prerequisites**: In this set of notes, I assume the reader is familiar with the programming concepts and Scala features covered in my *Notes on Scala for Java Programmers* [Cunningham 2019a], *Recursion Styles, Correctness, and Efficiency (Scala Version)* [Cunningham 2019b], *Type System Concepts* [Cunningham 2019c], *Functional Data Structures* [Cunningham 2019d], and *Handling Errors without Exceptions* [Cunningham 2019e].

**Browser Advisory:** The HTML version of this textbook requires a browser that supports the display of MathML. A good choice as of April 2019 is a recent version of Firefox from Mozilla.

## Strictness and Laziness

## Introduction

The big idea we discuss in this chapter is how we can exploit nonstrict functions to increase efficiency, increase code reuse, and improve modularity in functional programs.

#### Motivation

In our discussion [Cunningham 2019d] of Chapter 3 of Functional Programming in Scala [Chiusano 2015], we examined purely functional data structures—in particular, the immutable, singly linked list.

We also examined the design and use of several bulk operations—such as map, filter, foldLeft, and foldRight. Each of these operations makes a pass over the input list and often constructs a new list for its output.

Consider the Scala expression

```
List(10,20,30,40,50).map(_/10).filter(_%2 == 1).map(_*100) that generates the result:
List(100, 300, 500)
```

The evaluation of the expression requires three passes through the list. However, we could code a specialized function that does the same work in one pass.

Note: In this chapter, we use the method-chaining formulation of List from the standard library, not the one we developed in the "Functional Data Structures" chapter. :: constructs a list with its left operand as the head value and its right operand as the tail list.

It would be convenient if we could instead get a result similar to mfm by composing simpler functions like map and filter.

Can we do this?

We can by taking advantage of *nonstrict* functions to build a *lazy* list structure. We introduced the concepts of strict and nonstrict functions in Chapter 4 [Cunningham 2019e]; we elaborate on them in this chapter.

#### What are strictness and nonstrictness?

If the evaluation of an expression runs forever or throws an exception instead of returning an explicit value, we say the expression does not terminate—or that it evaluates to bottom (written symbolically as  $\bot$ ).

A function f is *strict* if f(x) evaluates to bottom for all x that themselves evaluate to bottom. That is,  $f(\bot) == \bot$ . A strict function's argument must always have a value for the function to have a value.

A function is *nonstrict* (sometimes called *lenient*) if it is not strict. That is,  $f(\bot) != \bot$ . The function can sometimes have value even if its argument does not have a value.

For multiparameter functions, we sometimes apply these terms to individual parameters. A *strict* parameter of a function must always be evaluated by the function. A *nonstrict* parameter of a function may sometimes be evaluated by the function and sometimes not.

### Exploring nonstrictness

By default, Scala functions are strict.

However, some operations are nonstrict. For example, the "short-circuited" && operation is nonstrict; it does not evaluate its second operand when the first operation is false. Similarly, || does not evaluate its second operand when its first operand is true.

Consider the if expression as a ternary operator. When the condition operand evaluates to true, the operator evaluates the second (i.e. then) operand but not the third (i.e. else) operand. Similarly, when the condition is false, the operator evaluates the third operand but not the second.

We could implement if as a function as follows:

Then we can call if 2 as in the code fragment

and get the output:

```
Can vote
```

The parameter type ()  $\Rightarrow$  A means that the corresponding argument is passed as a parameterless function that returns a value of type A. This function wraps the

expression, which is not evaluated before the call. This function is an explicitly specified *thunk*.

When the value is needed, then the called function must *force* the evaluation of the thunk by calling it explicitly, for example by using onTrue().

To use the approach above, the caller must explicitly create the thunk. However, as we saw in the previous chapter, Scala provides *call-by-name* parameter passing that relieves the caller of this requirement in most circumstances. We can thus rewrite if2 as follows:

```
def if2[A](cond: Boolean, onTrue: => A, onFalse: => A): A =
   if (cond) onTrue else onFalse
```

The onTrue: => A notation makes the argument expression a by-name parameter. Scala automatically creates the thunk for parameter onTrue and enables it to be referenced within the function without explicitly forcing its evaluation, for example by using onTrue.

An advantage of call-by-name parameter passing is that the evaluation of an expression can be delayed until its value is referenced, which may be never. A disadvantage is that the expression will be evaluated every time it is referenced.

To determine how to address this disadvantage, consider function

```
def maybeTwice(b: Boolean, i: => Int) = if (b) i + i else 0
which can be called as
    println(maybeTwice(true, {println("hi"); 1 + 41}))

to generate output:
    hi
    hi
    84
```

Note that the argument expression i is evaluated twice.

We can address this issue by defining a new variable and initializing it *lazily* to have the same value as the by-name parameter. We do this by declaring the temporary variable as a lazy val. The temporary variable will not be initialized until it is referenced, but it *caches* the calculated value so that it can be used without reevaluation on subsequent references.

We can rewrite maybeTwice as follows:

```
def maybeTwice2(b: Boolean, i: => Int) = {
    lazy val j = i
    if (b) j+j else 0
}
```

Now calling it as

```
println(maybeTwice2(true, {println("hi"); 1 + 41}))
generates output:
   hi
   84
```

This technique of caching the result of the evaluation gives us *call-by-need* parameter passing as it is called in Haskell and other lazily evaluated languages.

## Lazy Lists

Now let's return to the problem discussed in the Motivation subsection. How can we use laziness to improve efficiency and modularity of our programs?

In this section, we answer this question by developing *lazy lists* or *streams*. These allow us to carry out multiple operations on a list without always making multiple passes over the elements.

Consider a simple "stream" algebraic data type StreamC. A nonempty stream consists of a head and a tail, both of which must be nonstrict.

Note: The Functional Programming in Scala book uses algebraic data type Stream, which differs from the implementation of the similar Stream type in Scala's standard library. To avoid conflicts with the standard library type, these notes use StreamC.

For technical reasons, Scala does not allow by-name parameters in the constructors for case classes. Thus these components must be explicitly defined thunks whose evaluations are explicitly forced when their values are required.

```
import StreamC._
sealed trait StreamC[+A]
case object Empty extends StreamC[Nothing]
case class Cons[+A](h: () => A, t: () => StreamC[A])
    extends StreamC[A]

object StreamC {
    def cons[A](hd: => A, tl: => StreamC[A]): StreamC[A] = {
        lazy val head = hd // cache values once computed
        lazy val tail = tl
        Cons(() => head, () => tail) // create thunks for Cons
}
def empty[A]: StreamC[A] = Empty
def apply[A](as: A*): StreamC[A] =
    if (as.isEmpty)
        empty
    else
```

```
cons(as.head, apply(as.tail: _*))
}
```

#### Smart constructors and memoized streams

In the StreamC data type, we define two *smart constructors* to create new streams. By convention, these are functions defined in the companion object with the same names as the corresponding type constructors except they begin with a lowercase letter. They construct a data type object, ensuring that the needed integrity invariants are established. In the StreamC type, these take care of the routine work of creating the thunks, caching the values, and enabling transparent use of the parameters.

Smart constructor function cons takes the head and tail of the new StreamC as by-name parameters, equates these to lazy variables to cache their values, and then creates a Cons cell. The h and t fields of the Cons are explicitly defined thunks wrapping the head and the tail of the stream, respectively.

The evaluation of the thunk h of a Cons cell returns the value of the lazy variable head in the cell's closure. If this is the first access to head, then the access causes the corresponding by-name argument hd to be evaluated and cached in head. Subsequent evaluations of h get the cached value.

The evaluation of the thunk t of a Cons cell causes similar effects on the lazy variable tail and the corresponding by-name argument tl. However, the value of this argument is itself a StreamC, which may include lazily evaluated fields.

Smart constructor function empty just creates an Empty StreamC.

We define both smart constructors to have return type StreamC[A]. In addition to establishing the needed invariants, the use of the smart constructors helps Scala's type inference mechanism infer the StreamC type (which is what we usually want) instead of the subtypes associated with the case class/object constructors (which is what often will be inferred in Scala's object-oriented type system).

Convenience function apply takes a sequence of zero or more arguments and creates the corresponding StreamC.

If a function examines or traverses a StreamC, it must explicitly force evaluation of the thunks. In general, we should encapsulate such accesses within functions defined as a part of the StreamC implementation. (That is, we should practice *information hiding*, hide this design detail as a *secret* of the StreamC implementation as we discuss in the notes on abstract data types [Cunningham 2019f].)

An example of this is function headOption that optionally extracts the head of the stream.

```
def headOption: Option[A] = this match {
   case Empty => None
   case Cons(h,t) => Some(h()) // force thunk
}
```

It explicitly forces evaluation of the thunk and thus enables code that called it to work with the values.

This technique for caching the value of the by-name argument is an example of memoizing the function. In general, *memoization* is an implementation technique in which a function stores the return value computed for certain arguments. Instead of recomputing the value on a subsequent call, the function just returns the cached values. This technique uses memory space to (potentially) save computation time later.

### Helper functions

Now let's define a few functions that help us manipulate streams. We implement these as methods on the StreamC trait.

First, let's define a function toList that takes a StreamC (as its implicit argument) and constructs the corresponding Scala List. A standard backward recursive method can be defined as follows:

```
def toListRecursive: List[A] = this match {
   case Cons(h,t) => h() :: t().toListRecursive // force thunks
   case _ => List()
}
```

Of course, this method may suffer from stack overflow for long streams. We can remedy this by using a tail recursive auxiliary function that uses an accumulator to build up the list in reverse order and then reverses the constructed list.

```
def toList: List[A] = {
    @annotation.tailrec
    def go(s: StreamC[A], acc: List[A]): List[A] = s match {
        case Cons(h,t) => go(t(), h() :: acc) // force thunks
        case _ => acc
    }
    go(this, List()).reverse
}
```

To avoid the reverse, we could instead build up the list in a mutable ListBuffer using a loop and then, when finished, convert the buffer to an immutable List. We preserve the *purity* of the toList function by encapsulating use of the mutable buffer inside the function.

```
def toListFast: List[A] = {
   val buf = new collection.mutable.ListBuffer[A]
```

```
Qannotation.tailrec
def go(s: StreamC[A]): List[A] = s match {
    case Cons(h,t) =>
        buf += h() // force head thunk, add to end of buffer
        go(t()) // force tail thunk, process recursively
    case _ => buf.toList // convert buffer to immutable list
}
go(this)
}
```

Next, let's define function take to return the first n elements from a StreamC and function drop to skip the first n elements.

We can define method take using a standard backward recursive form that matches on the structure of the implicit argument. However, we must be careful not to evaluate either the head or the tail thunks unnecessarily (e.g. by treating the n == 1 and n == 0 cases specially).

```
def take(n: Int): StreamC[A] = this match {
   case Cons(h, t) if n > 1 => cons(h(), t().take(n - 1))
   case Cons(h, _) if n == 1 => cons(h(), empty)
   case _ => empty // stream empty or n < 1
}</pre>
```

Function take does its work *incrementally*. The recursive leg of the definition (i.e. the first leg) returns a Cons cell with the recursive call to take embedded in the lazily evaluated tail field. It will only be evaluated if its value is required.

We can define method **drop** to recursively calling **drop** on the forced tail. This yields the following tail recursive function.

```
@annotation.tailrec
final def drop(n: Int): StreamC[A] = this match {
   case Cons(_, t) if n > 0 => t().drop(n - 1)
   case _ => this
}
```

Unlike take, drop is not incremental. The recursive call is not lazily evaluated.

Finally, let's also define method takeWhile to return all starting elements of the StreamC that satisfy the given predicate.

```
def takeWhile(p: A => Boolean): StreamC[A] = this match {
   case Cons(h,t) if p(h()) => cons(h(), t() takeWhile p)
   case _ => empty
}
```

In the first case, we apply method takeWhile as an infix operator.

## Separating Program Description from Evaluation

One of the fundamental design concepts in software engineering and programming is *separation of concerns*. A concern is some set of information that affects the design and implementation of a software system [Wikipedia 2019a]. We identify the key concerns in a software design and try to keep them separate and independent from each other. The goal is to implement the parts independently and then combine the parts to form a complete solution.

We apply separation of concerns in modular programming and abstract data types as *information hiding* [Cunningham 2019f] [Parnas 1972] [Wikipedia 2019b]. We hide the *secrets* of how a module is implemented (e.g. what algorithms and data structures are used, what specific operating system or hardware devices are used, etc.) from the external users of the module or data type. We encapsulate the secrets behind an *abstract interface* [Britton 1981] [Cunningham 2019f].

We also apply separation of concerns in software architecture for computing applications. For example, we try to keep an application's business logic (i.e. specific knowledge about the application area) separate from its user interface such as described by the Model-View-Controller (MVC) architectural design pattern [Wikipedia 2019c] commonly used in Web applications.

In functional programming, we also apply separation of concerns by seeking to keep the description of computations separate from their evaluation (execution). Examples include:

- first-class functions that express computations in their bodies but which must be supplied arguments before they execute
- use of Option or Either to express that an error has occurred but deferring the handling of the error to other parts of the program
- use of StreamC operators to assemble a computation that generates a sequence without actually running the computation until later when its result in needed

## Laziness promotes reuse

In general, lazy evaluation enables us to separate the description of an expression from the evaluation of the expression. It enables us to to describe a "larger" expression than we need and then to only evaluate the portion that we actually need. This offers us the potential for greater *code reuse*.

Note: For a classic discussion of how higher-order and first-class functions and lazy evaluation promote software modularity and reuse, see the John Hughes paper "Why Functional Programming Matters" [Hughes 1989].

Consider a method exists on StreamC that checks whether an element matching a Boolean function p occurs in the stream. We can define this using an explicit

tail recursion as follows:

```
def exists(p: A => Boolean): Boolean = this match {
   case Cons(h,t) => p(h()) || t().exists(p)
   case _ => false
}
```

Given that | | is nonstrict in its second argument, this function terminates and returns true as soon as it finds the first element that makes p true. Because the stream holds the tail in a lazy val, it is only evaluated when needed. So exists does not evaluate the stream past the first occurrence.

As with the List data type in Chapter 3, we can define a more general method foldRight on StreamC to represent the pattern of computation exhibited by exists.

```
def foldRight[B](z: => B)(f: (A, => B) => B): B = this match {
    case Cons(h,t) => f(h(), t().foldRight(z)(f))
    case _ => z
}
```

The notation => B in the second parameter of combining function f takes its second argument by-name and, hence, may not evaluate it in all circumstances. If f does not evaluate its second argument, then the recursion terminates. Thus the overall foldRight computation can terminate before it completes the complete traversal through the stream.

We can now redefine exists to use the more general function as follows:

```
def exists2(p: A => Boolean): Boolean =
   foldRight(false)((a, b) => p(a) || b)
```

Here parameter b represents the unevaluated recursive step that folds the tail of the stream. If p(a) returns true, then b is not evaluated and the computation terminates early.

Caveat: The second version of exists illustrates how we can use a general function to represent a variety of more specific computations. But, for a large stream in which all elements evaluate to false, this version is not stack safe.

Because the foldRight method on StreamC can terminate its traversal early, we can use it to implement exists efficiently. Unfortunately, we cannot implement the List version of exists efficiently in terms of the List version of foldRight. We must implement a specialized recursive version of exists to get early termination.

Laziness thus enhances our ability to reuse code.

### Incremental computations

Now, let's flesh out the StreamC trait and implement the basic map, filter, append, and flatMap methods using the general function foldRight, as follows:

```
def map[B](f: A => B): StreamC[B] =
   foldRight(empty[B])((h,t) => cons(f(h), t))

def filter(p: A => Boolean): StreamC[A] =
   foldRight(empty[A])((h,t) => if (p(h)) cons(h, t) else t)

def append[B >: A](s: => StreamC[B]): StreamC[B] =
   foldRight(s)((h,t) => cons(h,t))

def flatMap[B](f: A => StreamC[B]): StreamC[B] =
   foldRight(empty[B])((h,t) => f(h) append t)
```

These implementations are *incremental*. They do not fully generate all their answers. No computation takes place until some other computation examines the elements of the output StreamC and then only enough elements are generated to give the requested result.

Because of their incremental nature, we can call these functions one after another without fully generating the intermediate results.

We can now address the problem raised in the Introduction section of these notes. There we asked the question of how can we compute the result of the expression

```
List(10,20,30,40,50).map(_/10).filter(_%2 == 1).map(_*100)
```

without producing two unneeded intermediate lists.

The StreamC expression

```
StreamC(10,20,30,40,50).map(_/10).filter(_%2 == 1).
map(_*100).toList
```

generates the result:

```
List(100, 300, 500)
```

which is the same as the List expression. The expression looks the same except that we create a StreamC initially instead of a List and we call toList to force evaluation of stream at the end.

When executed, the lazy evaluation interleaves two map, the filter, and the toList transformations. The computation does not fully instantiate any intermediate streams. It is a similar interleaving to what we did in the special purpose function mfm in the introduction.

(For a more detailed discussion of this interleaving, see Listing 5.3 in the Functional Programming in Scala book [Chiusano 2015].)

Because stream computations do not generate intermediate streams in full, we are free to use stream operations in ways that might seem counterintuitive at first. For example, we can use filter (which seems to process the entire stream) to implement find, a function to return the first occurrence of an element in a stream that satisfies a given predicate, as follows:

```
def find(p: A => Boolean): Option[A] = filter(p).headOption
```

The incremental nature of these computations can sometimes save memory. The computation may only need a small amount of working memory; the garbage collector can quickly recover working memory that the current step does not need.

Of course, some computations may require more intermediate elements and each element may itself require a large amount of memory, so not all computations are as well-behaved as the examples in this section.

### For comprehensions on streams

Given that we have defined map, filter, and flatMap, we can now use sequence comprehensions on our StreamC data. For example, the code fragment

causes the following to print on the console:

```
List(3, 3, 4, 4)
```

Note: During compilation, some versions of the Scala compiler may issue a deprecation warning that filter is used instead of withFilter. In a future release of Scala, this substitution may no longer work. Because filter is lazy for streams, we could define f<ilter as an alias for withFilter with the following:

```
def withFilter = filter _
```

However, filter does generate a new StreamC where withFilter normally does not generate a new collection. Although this gets rid of the warning, it would be better to implement a proper withFilter function.

#### Infinite Streams and Corecursion

Because the streams are incremental, the functions we have defined also work for *infinite streams*.

Consider the following definition for an infinite sequence of ones:

```
lazy val ones: StreamC[Int] = cons(1, ones)
```

Note: The book Functional Programming in Scala [Chuisano 2015] does not add the lazy annotation, but that version gives a compilation error in some versions of Scala. Adding lazy seems to fix the problem, but this issue should be investigated further.

Although ones is infinite, the StreamC functions only reference the finite prefix of the stream needed to compute the needed result.

For example:

```
ones.take(5).toList yields List(1,1,1,1,1)
ones.map(_+2).take(5).toList yields List(3,3,3,3,3)
What about ones.map(_+2).toList?
```

We can generalize ones to a constant function as follows:

```
def constant[A](a: A): StreamC[A] = {
   lazy val tail: StreamC[A] = Cons(() => a, () => tail)
   tail
}
```

An alternative would be just to make the body cons(a, constant(a)). But the above is more efficient because it is just one object referencing itself.

We can also define an increasing StreamC of all integers beginning with n as follows:

```
def from(n: Int): StreamC[Int] =
   cons(n, from(n+1))
```

The (second-order) Fibonacci sequence begins with the elements 0 and 1; each subsequent element is the sum of the two previous elements. We can define the Fibonacci sequence as a stream fibs with the following definition:

```
val fibs = {
    def go(f0: Int, f1: Int): StreamC[Int] =
        cons(f0, go(f1, f0+f1))
    go(0, 1)
}
```

#### Prime numbers: Sieve of Erastosthenes

A positive integer greater than 1 is *prime* if it is divisible only by itself and 1. The *Sieve of Eratosthenes* algorithm works by removing multiples of numbers once they are identified as prime.

- We begin the increasing stream of integers starting with 2, a prime number.
- The head is 2, so we remove all the multiples of 2 from the stream.

- The head of the tail is 3, so it is prime because it was not removed as a multiple of 2 and it is the smallest integer remaining.
- Continue the process recursively on the tail.

We can define this calculation with the following StreamC functions.

```
def sieve(ints: StreamC[Int]): StreamC[Int] =
  ints.headOption match {
    case None =>
        sys.error(
        "Should not occur: No head on infinite stream.")
    case Some(x) =>
        cons(x,sieve(ints drop 1 filter (_ % x > 0)))
  }
val primes: StreamC[Int] = sieve(from(2))
```

We can then use **primes** to define a function **isPrime** to test whether an integer is prime.

#### Function unfold

Now let's consider unfold, a more general stream-building function. Function unfold takes an initial state and a function that produces both the next state and the next value in the stream and builds the resulting stream. We can define it as follows:

```
def unfold[A, S](z: S)(f: S => Option[(A, S)]): StreamC[A] =
   f(z) match {
      case Some((h,s)) => cons(h, unfold(s)(f))
      case None => empty
}
```

This function applies f to the current state z to generate the next state s and the next element h of the stream. We use Option so f can signal when to terminate the StreamC.

Function unfold is an example of a corecursive function.

A recursive function consumes data. The input of each successive call is "smaller" than the previous one. Eventually the recursion terminates when input size reaches the minimum.

A corecursive function produces data. Corecursive functions need not terminate as long as they remain productive. By productive, we mean that the function

can continue to evaluate more of the result in a finite amount of time.

Where we seek to argue that recursive functions terminate, we seek to argue that corecursive functions are productive.

The unfold function remains productive as long as its argument function f terminates. Function f must terminate for the unfold computation to reach its next state.

Some writers in the functional programming community use the term guarded recursion instead of corecursion and the term cotermination instead of productivity. See the Wikipedia articles on corecursion [Wikipedia 2019d] and coinduction [Wikipedia 2019e] for more information and links.

The function unfold is very general. For example, we can now define ones, constant, from, and fibs with unfold.

```
val onesViaUnfold = unfold(1)(_ => Some((1,1)))

def constantViaUnfold[A](a: A) =
    unfold(a)(_ => Some((a,a)))

def fromViaUnfold(n: Int) =
    unfold(n)(n => Some((n,n+1)))

val fibsViaUnfold =
    unfold((0,1)) { case (f0,f1) => Some((f0,(f1,f0+f1))) }
```

## Summary

The big idea in this chapter is that we can exploit nonstrict functions to increase efficiency, increase code reuse, and improve the modularity in functional programs.

## Source Code for Chapter

• StreamC.scala

#### **Exercises**

TODO: Add

## Acknowledgements

In Spring 2016, I wrote this set of notes to accompany my lectures on Chapter 5 of the book *Functional Programming in Scala* [Chiusano 2015] (i.e. the Red Book). I constructed the notes around the ideas, general structure, and Scala examples from that chapter and its associated materials.

In 2018 and 2019, I updated the format of the document to be more compatible with my evolving document structures. In 2019, I also renamed the Stream (used in the Red Book) to StreamC to better avoid conflicts with the standard library type Stream.

I maintain these notes as text in Pandoc's dialect of Markdown using embedded LaTeX markup for the mathematical formulas and then translate the notes to HTML, PDF, and other forms as needed.

### References

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## Terms and Concepts

 $Big\ idea$ : Exploiting nonstrict function to increase efficiency, increase code reuse, and improve modularity

Concepts: Strict and nonstrict (lenient) functions/parameters, termination, bottom, call-by-name, thunk, forcing, call-by-need, lazy evaluation, lazy lists or streams, Stream data type, smart constructors, memoization, lazy variables, purity of functions, separation of concerns, information hiding, design secret, abstract interface, business logic, Model-View-Controller (MVC) design pattern, keeping program description separate from evaluation, incremental computation, prime number, Sieve of Eratosthenes, recursive, corecursive (guarded recursion), productivity (cotermination).