CSci 550 : Program Semantics & Derivation
Spring Semester 2006, Assignment #4
Due Tuesday, 11 April, 8:00 A.M. (Extended)

1. Prove that the following program is correct. Identify the proof obligations and show that they hold.

$$
\begin{array}{l}
|[\ \ \textbf{var}\ c, n : int; \textbf{var}\ b[0..N) : \textbf{array of}\ int\ \{0 < n \leq N \wedge c = (\#\ i : n \leq i < N : b.i < 0)\}\ ; \\
\quad \textbf{if}\ \ b.(n-1) < 0\ \ \rightarrow c, n := c+1, n-1 \\
\quad [\!]\ \ b.(n-1) \geq 0\ \ \rightarrow n := n-1 \\
\quad \textbf{fi} \\
\quad \{c = (\#\ i : n \leq i < N : b.i < 0)\} \\
]|
\end{array}
$$

2. Now prove that the following program is correct. Find a loop invariant and bound function for the loop. Identify the proof obligations and show that they hold.

$$
\begin{array}{l}
|[\ \ \textbf{var}\ c, n : int; \textbf{var}\ b[0..N) : \textbf{array of}\ int\{N \geq 0\}\ ; \\
\quad c, n := 0, N; \\
\quad \textbf{do}\ n \neq 0 \rightarrow \\
\qquad \textbf{if}\ \ b.(n-1) < 0 \rightarrow c, n := c+1, n-1 \\
\qquad [\!]\ \ b.(n-1) \geq 0 \rightarrow n := n-1 \\
\qquad \textbf{fi} \\
\quad \textbf{od} \\
\quad \{c = (\#\ i : 0 \leq i < N : b.i < 0)\} \\
]|
\end{array}
$$

3. How would the invariant and bound function differ for this program? It is not necessary to prove this program, just identify a suitable invariant and bound function.

$$
\begin{array}{l}
|[\ \ \textbf{var}\ c, n : int; \textbf{var}\ b[0..N) : \textbf{array of}\ int\{N \geq 0\}\ ; \\
\quad c, n := 0, 0; \\
\quad \textbf{do}\ n \neq N \rightarrow \\
\qquad \textbf{if}\ \ b.n < 0 \rightarrow c, n := c+1, n+1 \\
\qquad [\!]\ \ b.n \geq 0 \rightarrow n := n+1 \\
\qquad \textbf{fi} \\
\quad \textbf{od} \\
\quad \{c = (\#\ i : 0 \leq i < N : b.i < 0)\} \\
]|
\end{array}
$$

4. Prove that the following program is correct. Find a loop invariant and bound function for the loop. Identify the proof obligations and show that they hold.

$$
\begin{array}{l}
|[\ \ \textbf{con}\ N : int\{N \geq 1\}\ ; \textbf{var}\ n : int\ ; \\
\quad n := 1; \\
\quad \textbf{do}\ 2 * n \leq N \rightarrow n := 2 * n\ \textbf{od} \\
\quad \{1 \leq n \leq N < 2 * n \wedge (\exists p : p \geq 0 : n = 2^p)\} \\
]|
\end{array}
$$