1. Prove that the following program is correct. Identify the proof obligations and show that they hold.

\[
\text{var} \ c, n : \text{int}; \ \text{array of} \ b[0..N] : \text{int} \ \{0 < n \leq N \land c = (\# \ i : n \leq i < N : b.i < 0)\};
\]

\[\text{if} \ b.(n - 1) < 0 \rightarrow c, n := c + 1, n - 1;\]
\[\text{if} \ b.(n - 1) \geq 0 \rightarrow n := n - 1;\]
\[\{c = (\# \ i : n \leq i < N : b.i < 0)\}\]

2. Now prove that the following program is correct. Find a loop invariant and bound function for the loop. Identify the proof obligations and show that they hold.

\[
\text{var} \ c, n : \text{int}; \ \text{array of} \ b[0..N] : \text{int} \ \{N \geq 0\};
\]

\[c, n := 0, N;\]
\[\text{do} \ n \neq 0 \rightarrow \text{if} \ b.(n - 1) < 0 \rightarrow c, n := c + 1, n - 1;\]
\[\text{if} \ b.(n - 1) \geq 0 \rightarrow n := n - 1;\]
\[\text{od} \{c = (\# \ i : 0 \leq i < N : b.i < 0)\}\]

3. How would the invariant and bound function differ for this program? It is not necessary to prove this program, just identify a suitable invariant and bound function.

\[
\text{var} \ c, n : \text{int}; \ \text{array of} \ b[0..N] : \text{int} \ \{N \geq 0\};
\]

\[c, n := 0, 0;\]
\[\text{do} \ n \neq N \rightarrow \text{if} \ b.n < 0 \rightarrow c, n := c + 1, n + 1;\]
\[\text{if} \ b.n \geq 0 \rightarrow n := n + 1;\]
\[\text{od} \{c = (\# \ i : 0 \leq i < N : b.i < 0)\}\]

4. Prove that the following program is correct. Find a loop invariant and bound function for the loop. Identify the proof obligations and show that they hold.

\[
\text{con} \ N : \text{int} \ \{N \geq 1\}; \ \text{var} \ n : \text{int};
\]
\[n := 1;\]
\[\text{do} \ 2 \ast n \leq N \rightarrow n := 2 \ast n;\]
\[\{1 \leq n \leq N \land (\exists p : p \geq 0 : n = 2^p)\}\]