

CSci 550 : Program Semantics & Derivation
Spring Semester 2006, Assignment #2
Due Thursday, 16 February, 8:00 A.M.

Do the following exercises.

1. Prove Theorem 62 from the Logic handout
2. Prove $X \equiv (X \neq Y) \equiv \neg Y$
3. Prove $(X \Rightarrow Z) \vee (Y \Rightarrow Z) \equiv X \wedge Y \Rightarrow Z$
4. Prove $Y \vee (X \Rightarrow Y) \equiv X \Rightarrow Y$
5. Prove Theorem 78 from the Logic handout.
6. Prove Theorem 94 from the Logic handout.
7. Given the validity of

- (a) $x = (\Sigma i : R.i : f.i)$
- (b) $R.i \neq i = N$ for any i

calculate a quantifier-free expression equivalent to

$$(\Sigma i : R.i \vee i = N : f.i).$$

The solution should take the form of our proofs with hints.

8. Given the validity of
- (a) $x = (\mathbf{MAX} i, j : R.i.j \wedge j < N + 1 : f.i.j)$
 - (b) $y = (\mathbf{MAX} i : R.i.(N + 1) : f.i.(N + 1))$

calculate a quantifier-free expression equivalent to

$$(\mathbf{MAX} i.j : R.i.j \wedge (j < N + 1 \vee j = N + 1) : f.i.j).$$

9. Given the validity of
- (a) $x = (\mathbf{MAX} i : R.i.N : f.i.N)$
 - (b) $R.y.(z + 1) \equiv R.y.z \vee y = z + 1$ for any y and z
 - (c) $f.y.y = 0$ for any y
 - (d) $f.y.(z + 1) = f.y.z + g.z$ for any y and z
 - (e) $R.y.z \neq false$ for any y and z

calculate a quantifier-free expression equivalent to

$$(\mathbf{MAX} i : R.i.(N + 1) : f.i.(N + 1)).$$