CSci 311 : Models of Computation
CSci 500 : Fundamental Concepts of Computing
Fall Semester 2000, Assignment #1
Due 11:00 A.M., Friday, 1 September 2000

1. Let $X = \{a, b, c, d\}$ and $Y = \{a, c, d, e\}$. Explicitly define the following sets:

    (a) $X \cup Y$

    (b) $X \cap Y$

    (c) $X - Y$

    (d) $Y - X$

    (e) 2^X

2. Let $X = \{2, 4, 6\}$ and $Y = \{a, b\}$.

    (a) List all the subsets of X.

    (b) List the members of $X \times Y$.

    (c) List all the total functions from Y to X.

3. **Assume** the following facts. Let Σ be an alphabet.

    (a) For all strings $w \in \Sigma^*$, $w\lambda = \lambda w = w$. (λ is the identity element for string concatenation.)

    (b) $\lambda^R = \lambda$.

    (c) For all symbols $a \in \Sigma$, $a^R = a$.

    (d) For all strings $w \in \Sigma^*$ and symbols $a \in \Sigma$, $(wa)^R = aw^R$.

    (e) For all strings $u \in \Sigma^*$ and symbols $v \in \Sigma^*$, $(uv)^R = v^R u^R$.

    (f) Basic properties of integer arithmetic such as associativity and commutativity of addition and multiplication, identity elements for addition (i.e., 0) and multiplication (i.e., 1), and distribution of multiplication over addition.

    Prove $(w^R)^R = w$ for all strings $w \in \Sigma^*$. Give justifications for each of your steps (e.g., facts from the above list).

4. Prove DeMorgan's Laws for sets:

    (a) $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$

    (b) $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$

5. Draw a picture of a graph that represents those states of the United States that touch the Mississippi River or touch states that do. Show an edge between states if they touch.