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Advisory: The HTML version of this document requires use of a browser that supports the display of MathML. A good choice as of July 2018 is a recent version of Firefox from Mozilla.
18 More List Processing

18.1 Chapter Introduction

TODO

Note: The source file for the code in this chapter is in MoreLists.hs.

18.2 Sequences

Haskell provides a compact notation for expressing arithmetic sequences.

An arithmetic sequence (or progression) is a sequence of elements from an enumerated type (i.e. a member of class Enum) such that consecutive elements have a fixed difference. Int, Integer, Float, Double, and Char are all predefined members of this class.

- \([\mathbf{\ldots}m\ldots n]\) produces the list of elements from \(m\) up to \(n\) in steps of one if \(m \leq n\). It produces the nil list otherwise.

Examples:
  - \([1\ldots5]\) \(\Rightarrow\) \([1,2,3,4,5]\)
  - \([5\ldots1]\) \(\Rightarrow\) \([\,]\)

This feature is implemented with Prelude function `enumFromTo` applied as `enumFromTo m n`.

- \([m,m'\ldots n]\) produces the list of elements from \(m\) in steps of \(m'-m\). If \(m' > m\) then the list is increasing up to \(n\). If \(m' < m\), then it is decreasing.

Examples:
  - \([1,3\ldots9]\) \(\Rightarrow\) \([1,3,5,7,9]\)
  - \([9,8\ldots5]\) \(\Rightarrow\) \([9,8,7,6,5]\)
  - \([9,8\ldots11]\) \(\Rightarrow\) \([\,]\)

This feature is implemented with Prelude function `enumFromThenTo` applied as `enumFromThenTo m' m n`.

- \([\mathbf{\ldots}]\) and \([m,m'\ldots]\) produce potentially infinite lists beginning with \(m\) and having steps 1 and \(m'-m\) respectively.

These features are implemented with Prelude functions `enumFrom` applied as `enumFrom m` and `enumFromThen` applied as `enumFromThen m m'`.

Of course, we can provide our own functions for sequences. Consider the following function to generate a geometric sequence.
A geometric sequence (or progression) is a sequence of elements from an ordered, numeric type (i.e., a member of both classes `Ord` and `Num`) such that consecutive elements have a fixed ratio.

```haskell
geometric :: (Ord a, Num a) => a -> a -> a -> [a]
geometric r m n | m > n = []
| otherwise = m : geometric r (m*r) n
```

Example: `geometric 2 1 10` \(\Rightarrow\) \([1,2,4,8]\)

### 18.3 List Comprehensions

#### 18.3.1 Syntax and semantics

The list comprehension is another powerful and compact notation for describing lists. A list comprehension has the form

\[
\{ \text{expression} \mid \text{qualifiers} \}
\]

where `expression` is any Haskell expression.

The `expression` and the `qualifiers` in a comprehension may contain variables that are local to the comprehension. The values of these variables are bound by the `qualifiers`.

For each group of values bound by the qualifiers, the comprehension generates an element of the list whose value is the `expression` with the values substituted for the local variables.

There are three kinds of `qualifiers` that can be used in Haskell: generators, filters, and local definitions.

1. A **generator** is a qualifier of the form

   \[ \text{pat} \leftarrow \text{exp} \]

   where `exp` is a list-valued expression. The generator extracts each element of `exp` that matches the pattern `pat` in the order that the elements appear in the list; elements that do not match the pattern are skipped.

   Example: \([ n*n \mid n \leftarrow [1..5]\] \(\Rightarrow\) \([1,4,9,16,25]\)

2. A **filter** is a Boolean-valued expression used as a `qualifier` in a list comprehension. These expressions work like the `filter` function; only values that make the expression `True` are used to form elements of the list comprehension.

   Example:

   \[ [ n*n \mid \text{even } n ] \Rightarrow (\text{if even } n \text{ then } [n*n] \text{ else } []) \]

   Above variable `n` is global to this expression, not local to the comprehension.
3. A *local definition* is a qualifier of the form

```
let pat = expr
```

introduces a local definition into the list comprehension.

Example:
```
• [ n*n | let n = 2 ] \implies [4]
```

The real power of list comprehensions come from using several qualifiers separated by commas on the right side of the vertical bar `|`.

- Generators appearing later in the list of qualifiers vary more quickly than those that appear earlier. Speaking operationally, the generation “loop” for the later generator is nested within the “loop” for the earlier.

Example:
```
• [ (m,n) | m<-[1..3], n<-[4,5] ] \implies [(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)]
```

- Qualifiers appearing later in the list of qualifiers may use values generated by qualifiers appearing earlier, but not vice versa.

Examples:
```
– [ n*n | n<-[1..10], even n ] \implies [4,16,36,64,100]
– [ (m,n) | m<-[1..3], n<-[1..m] ] \implies [(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)]
```

- The generated values may or may not be used in the *expression*.

Examples:
```
– [ 27 | n<-[1..3]] \implies [27,27,27]
– [ x | x<-[1..3], y<-[1..2]] \implies [1,1,2,2,3,3]
```

### 18.3.2 Translating list comprehensions

List comprehensions are syntactic sugar. We can translate them into core Haskell features by applying the following identities.

1. For any expression $e$,

```
[ e | True ]
```

is equivalent to:

```
[ e ]
```

2. For any expression $e$ and qualifier $q$,

```
[ e | q ]
```
is equivalent to:

\[ [ e \mid q, \text{True} ] \]

3. For any expression \( e \), boolean \( b \), and sequence of qualifiers \( Q \),

\[ [ e \mid b, Q ] \]

is equivalent to:

\[
\text{if } b \text{ then } [ e \mid Q ] \text{ else } []
\]

4. For any expression \( e \), pattern \( p \), list-valued expression \( l \), sequence of qualifiers \( Q \), and fresh variable \( \text{ok} \),

\[ [ e \mid p \leftarrow l, Q ] \]

is equivalent to:

\[
\text{let } \text{ok } p = [ e \mid Q ] \quad -- \quad p \text{ is a pattern}
\text{ok } _ = []
\text{in } \text{concatMap } \text{ok } l
\]

5. For any expression \( e \), declaration list \( D \), and sequence of qualifiers \( Q \),

\[ [ e \mid \text{let } D, Q ] \]

is equivalent to:

\[
\text{let } D \text{ in } [ e \mid Q ]
\]

Function \text{concatMap} and boolean value \text{True} are as defined in the Prelude.

As we saw in a previous chapter, \text{concatMap} applies a list-returning function to each element of an input list and then concatenates the resulting list of lists into a single list. Both \text{map} and \text{filter} can be defined in terms of \text{concatMap}.

Consider the list comprehension:

\[ [ n^2 \mid n \leftarrow \{1\ldots10\}, \text{even } n ] \]

a. Apply identity 4:

\[
\text{let } \text{ok } n = [ n^2 \mid \text{even } n ]
\text{ok } _ = []
\text{in } \text{concatMap } \text{ok } [1..10]
\]

b. Apply identity 2:

\[
\text{let } \text{ok } n = [ n^2 \mid \text{even } n, \text{True} ]
\text{ok } _ = []
\text{in } \text{concatMap } \text{ok } [1..10]
\]

c. Apply identity 3:
let ok n = if (even n) then [ n*n | True ]
  ok _ = []
in concatMap ok [1..10]

18.3.3 Strings of spaces

Consider a function \( \text{spaces} \) that takes a number and generates a string with that many spaces.

\[
\text{spaces} :: \text{Int} \to \text{String}
\]

\[
\text{spaces} n = [' ' | i<-[1..n]]
\]

Note that when \( n < 1 \) the result is the empty string.

18.3.4 Prime number test

Consider a Boolean function \( \text{isPrime} \) that takes a nonzero natural number and determines whether the number is prime. (Remember that a prime number is a natural number whose only natural number factors are 1 and itself.)

\[
\text{isPrime} :: \text{Int} \to \text{Bool}
\]

\[
\text{isPrime} n | n > 1 = (\text{factors} n == [])
\]

\[
\text{where factors} m = [ x | x<-[2..(m-1)], m \mod x == 0 ]
\]

\[
\text{isPrime} _ = \text{False}
\]

18.3.5 Squares of primes

Consider a function \( \text{sqPrimes} \) that takes two natural numbers and returns the list of squares of the prime numbers in the inclusive range from the first up to the second.

\[
\text{sqPrimes} :: \text{Int} \to \text{Int} \to [\text{Int}]
\]

\[
\text{sqPrimes} m n = [ x*x | x<-[m..n], \text{isPrime} x ]
\]

Alternatively, this function could be defined using \( \text{map} \) and \( \text{filter} \) as follows:

\[
\text{sqPrimes'} :: \text{Int} \to \text{Int} \to [\text{Int}]
\]

\[
\text{sqPrimes'} m n = \text{map} (\lambda x \to x*x) \text{ (filter isPrime [m..n])}
\]
18.3.6 Doubling positive elements

We can use a list comprehension to define (our, by now, old and dear friend) the function \texttt{doublePos}, which doubles the positive integers in a list.

\begin{verbatim}
doublePos5 :: [Int] -> [Int]
doublePos5 xs = [ 2*x | x<-xs, 0 < x ]
\end{verbatim}

18.3.7 Concatenating a list of lists of lists

Consider a program \texttt{superConcat} that takes a list of lists of lists and concatenates the elements into a single list.

\begin{verbatim}
superConcat :: [[[a]]] -> [a]
superConcat xsss = [ x | xss<-xsss, xs<-xss, x<-xs ]
\end{verbatim}

Alternatively, this function could be defined using Prelude functions \texttt{concat} and \texttt{map} and functional composition as follows:

\begin{verbatim}
superConcat' :: [[[a]]] -> [a]
superConcat' = concat . map concat
\end{verbatim}

18.3.8 First occurrence in a list

Consider a function \texttt{position} that takes a list and a value of the same type. If the value occurs in the list, \texttt{position} returns the position of the value’s first occurrence; if the value does not occur in the list, \texttt{position} returns 0.

\textbf{Strategy:} Solve a more general problem first, then use it to get the specific solution desired.

In this problem, we generalize the problem to finding all occurrences of a value in a list. This more general problem is actually easier to solve.

\begin{verbatim}
positions :: Eq a => [a] -> a -> [Int]
positions xs x = [ i | (i,y)<-zip [1..length xs] xs, x == y]
\end{verbatim}

Function \texttt{zip} is useful in pairing an element of the list with its position within the list. The subsequent filter removes those pairs not involving the value \texttt{x}. The “zipper” functions can be very useful within list comprehensions.

Now that we have the positions of all the occurrences, we can use \texttt{head} to get the first occurrence. Of course, we need to be careful that we return 0 when there are no occurrences of \texttt{x} in \texttt{xs}.

\begin{verbatim}
position :: Eq a => [a] -> a -> Int
position xs x = head ( positions xs x ++ [0] )
\end{verbatim}
Because of lazy evaluation, this implementation of `position` is not as inefficient
as it first appears. The function `positions` will, in actuality, only generate the
head element of its output list.

Also because of lazy evaluation, the upper bound `length xs` can be left off the
generator in `positions`. In fact, the function is more efficient to do so.

18.4 What Next?

TODO

18.5 Exercises

1. Show the list (or string) yielded by each of the following Haskell list expres-
sions. Display it using fully specified list bracket notation, e.g. expression
   `[1..5]` yields `[1,2,3,4,5].
   
a. `[7..11]`
b. `[11..7]`
c. `[3,6..12]`
d. `[12,9..2]`
e. `[n*n | n <- [1..10], even n]`
f. `[7 | n <- [1..4]]`
g. `[x | (x:xs) <- [Did, you, study?]]`
h. `[x,y | x <- [1..3], y <- [4,7]]`
i. `[m,n | m <- [1..3], n <- [1..m]]`
j. `take 3 [ [1..n] | n <- [1..] ]`

2. Translate the following expressions into expressions that use list compre-
hensions. For example, `map (*2) xs` could be translated to `[x*2 | x <- xs].
   
a. `map (\x -> 2*x-1) xs`
b. `filter p xs`
c. `map (^2) (filter even [1..5])`
d. `foldr (++) [] xss`
e. `map snd (filter (p . fst) (zip xs [1..]))`
18.6 Acknowledgements

In 2016 and 2017, I adapted and revised my previous notes to form Chapter 7, More List Processing and Problem Solving, in the 2017 version of this textbook:

- chapter 7 of my Notes on Functional Programming with Haskell for discussion of sequences and list comprehensions in sections 7.2 and 7.3 [Cunningham 2014]:
- chapter 10 of my Notes on Functional Programming with Haskell for discussion of problem-solving techniques in section 7.4
- part of chapter 12 of Notes on Functional Programming with Haskell for discussion of the tupling technique incorporated into subsection 7.4.2.1 [Cunningham 2014]:

In Summer 2018, I divided the previous More List Processing and Problem Solving chapter back into two chapters in the 2018 version of the textbook, now titled Exploring Languages with Interpreters and Functional Programming. Previous sections 7.2-7.3 (essentially the first item above) became the basis for new Chapter 18 (this chapter), More List Processing, and previous section 7.4 (essentially the second and third items above) became the basis for new Chapter 20, Problem Solving.

I maintain this chapter as text in Pandoc’s dialect of Markdown using embedded LaTeX markup for the mathematical formulas and then translate the document to HTML, PDF, and other forms as needed.

18.7 References

TODO


18.8 Terms and Concepts

TODO