Exploring Languages with Interpreters and Functional Programming Chapter 6

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Contents

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6 Procedural Abstraction

6.1 Chapter Introduction

Chapter 2 introduced the concepts of procedural and data abstraction. This chapter focuses on procedural abstraction. The next chapter focuses on on data abstraction.

The goals of this chapter are to:

- illustrate use of procedural abstraction, in particular of the top-down, stepwise refinement approach to design
- introduce modular programming using Haskell modules

6.2 Using Top-Down Stepwise Refinement

A useful and intuitive design process for a small program is to begin with a high-level solution and incrementally fill in the details. We call this process top-down stepwise refinement. Here we introduce it with an example.

6.2.1 Developing a square root package

Consider the problem of computing the nonnegative square root of a nonnegative number *x*. Mathematically, we want to find the number *y* such that

$$
y \ge 0
$$
 and $y^2 = x$.

A common algorithm in mathematics for computing the above *y* is to use Newton's method of successive approximations, which has the following steps for square root:

- 1. Guess at the value of *y*.
- 2. If the current approximation (guess) is sufficiently close (i.e. good enough), return it and stop; otherwise, continue.
- 3. Compute an improved guess by averaging the value of the guess y and x/y , then go back to step 2.

To encode this algorithm in Haskell, we work top down to decompose the problem into smaller parts until each part can be solved easily. We begin this *top-down stepwise refinement* by defining a function with the type signature:

sqrtIter :: Double -> Double -> Double

We choose type Double (double precision floating point) to approximate the real numbers. Thus we can encode step 2 of the above algorithm as the following Haskell function definition:

```
sqrtIter guess x -- step 2
  | goodEnough guess x = guess
  | otherwise = sqrtIter (improve guess x) x
```
We define function sqrtIter to take two arguments—the current approximation guess and nonnegative number x for which we need the square root. We have two cases:

• When the current approximation guess is sufficiently close to x, we return guess.

We abstract this decision into a separate function goodEnough with type signature:

```
goodEnough :: Double -> Double -> Bool
```
• When the approximation is not yet close enough, we continue by reducing the problem to the application of sqrtIter itself to an improved approximation.

We abstract the improvement process into a separate function improve with type signature:

```
improve :: Double -> Double -> Double
```
To ensure termination of sqrtIter, the argument (improve guess x) on the recursive call must get closer to termination (i.e. to a value that satisfies its base case).

The function improve takes the current guess and x and carries out step 3 of the algorithm, thus averaging guess and x/guess, as follows:

```
improve :: Double -> Double -> Double -- step 3
improve guess x = average guess (x/guess)
```
Function application improve y x assumes $x \gt o 0$ & y > 0. We call this a *precondition* of the improve y x function.

Because of the precondition of improve, we need to strengthen the precondition of sqrtIter guess x to x >= 0 && guess > 0. $<$

In improve, we abstract average into a separate function as follows:

average :: Double -> Double -> Double average $x y = (x + y) / 2$

The new guess is closer to the square root than the previous guess. Thus the algorithm will terminate assuming a good choice for function goodEnough, which guards the base case of the sqrtIter recursion.

How should we define goodEnough? Given that we are working with the limited precision of computer floating point arithmetic, it is not easy to choose an appropriate test for all situations. Here we simplify this and use a tolerance of 0.001.

We thus postulate the following definition for goodEnough:

goodEnough :: Double -> Double -> Bool goodEnough guess $x = abs$ (square guess - x) < 0.001

In the above, abs is the built-in absolute value function defined in the standard Prelude library. We define square as the following simple function (but could replace it by just guess $*$ guess):

```
square :: Double -> Double
square x = x * x
```
What is a good initial guess? It is sufficient to just use 1. So we can define an overall square root function sqrt' as follows:

sqrt' :: Double -> Double sqrt' x | x >= 0 = sqrtIter 1 x

(A square root function sqrt is defined in the Prelude library, so a different name is needed to avoid the name clash.)

Function sqrt' x has precondition $x \ge 0$. This and the choice of 1 for the initial guess ensure that functions sqrtIter and improve are applied with arguments that satisfy their preconditions.

6.2.2 Making the package a Haskell module

We can make this package into a Haskell module by putting the definitions in a file (e.g. named Sqrt.hs) and adding a module header at the beginning as follows:

```
module Sqrt
    ( sqrt' )
where
    -- give the definitions above for functions sqrt',
    -- sqrtIter, improve, average, and goodEnough
```
The header gives the module the name Sqrt and defines the names in parenthesis as being *exported* from this module to other modules that *import* this module. The other symbols (e.g. sqrtIter, goodEnough, improve) are local to (i.e. hidden inside) the module.

We can import module Sqrt into a module such as TestSqrt shown below. The **import** makes all the definitions exported by Sqrt available with module TestSqrt.

```
module TestSqrt
where
import Sqrt -- file Sqrt.hs, import all public names
```

```
main = do
    putStrLn (show (sqrt' 16))
    putStrLn (show (sqrt' 2))
```
In the above Haskell code, the symbol "--" denotes the beginning of an endof-line comment. All text after that symbol is text ignored by the Haskell compiler.

The Haskell module for the Square root case study is in file <Sqrt.hs>. Limited testing code is in module [SqrtTest.hs](TestSqrt.hs).

6.2.3 Reviewing top-down stepwise refinement

The program design strategy known as *top-down stepwise refinement* is a relatively intuitive design process that has long been applied in the design of structured programs in imperative procedural languages. It is also useful in the functional setting.

In Haskell, we can apply top-down stepwise refinement as follows.

1. Start with a high-level solution to the problem consisting of one or more functions. For each function, identify its type signature and functional requirements (i.e. its inputs, outputs, and termination condition).

Some parts of each function may be abstracted as "pseudocode" expressions or as-yet-undefined functions.

2. Choose one of the incomplete parts. Consider the specified type signature and functional requirements. Refine the incomplete part by breaking it into subparts or, if simple, defining it directly in terms of Haskell expressions (including calls to the Prelude, other available library functions, or previously defined parts of the algorithm).

When refining an incomplete part, consider the various options according to the relevant design criteria (e.g., time, space, generality, understandability, elegance, etc.)

The refinement of the function may require a refinement of the data being passed. If so, back up in the refinement process and readdress previous design decisions as needed.

If it not possible to design an appropriate refinement, back up in the refinement process and readdress previous design decisions.

3. Continue step 2 until all parts are fully defined in terms of Haskell code and data and the resulting set of functions meets all required criteria.

For as long as possible, we should stay with terminology and notation that is close to the problem being solved. We can do this by choosing appropriate function names and signatures and data types. (In other chapters, we examine Haskell's rich set of builtin and user-defined types.)

For stepwise refinement to work well, we must be willing to back up to earlier design decisions when appropriate. We should keep good documentation of the intermediate design steps.

The stepwise refinement method can work well for small programs , but it may not scale well to large, long-lived, general purpose programs. In particular, stepwise refinement may lead to a module structure in which modules are tightly coupled and not robust with respect to changes in requirements.

A combination of techniques may be needed to develop larger software systems. In the next section, we consider the use of modular design techniques.

6.3 Modular Design and Programming

Software engineering pioneer David Parnas defines a *module* as "a work assignment given to a programmer or group of programmers" [Parnas 1978]. This is a *software engineering* view of a module.

In a programming language like Haskell, a **module** is also a program unit defined with a construct or convention. This is a *programming language* view of a module.

Ideally, a language's module features should support the software engineering module methods.

6.3.1 Information-hiding modules and secrets

According to Parnas, the goals of *modular design* are to [Parnas 1972]:

- 1. enable programmers to understand the system by focusing on one module at a time (i.e. *comprehensibility*).
- 2. shorten development time by minimizing required communication among groups (i.e. *independent development*).
- 3. make the software system flexible by limiting the 'number of modules affected by significant changes (i.e. *changeability*).

Parnas advocates the use of a principle he called *information hiding* to guide decomposition of a system into appropriate modules (i.e. work assignments). He points out that the connections among the modules should have as few information requirements as possible [Parnas 1972].

In the Parnas approach, an information-hiding module:

- forms a *cohesive* unit of functionality *separate* from other modules
- *hides* a design decision—its *secret*—from other modules

• *encapsulates* an aspect of system likely to change (its secret)

Aspects likely to change independently of each other should become secrets of separate modules. Aspects unlikely to change can become interactions (connections) among modules.

This approach supports the goal of changeability (goal 2). When care is taken to design the modules as clean abstractions with well-defined and documented interfaces, the approach also supports the goals of independent development (goal 1) and comprehensibility (goal 3).

Information hiding has been absorbed into the dogma of contemporary objectoriented programming. However, information hiding is often oversimplified as merely hiding the data and their representations [Weiss 2001].

The secret of a well-designed module may be much more than that. It may include such knowledge as a specific functional requirement stated in the requirements document, the processing algorithm used, the nature of external devices accessed, or even the presence or absence of other modules or programs in the system [Parnas 1972, 1979, 1985]. These are important aspects that may change as the system evolves.

6.3.1.1 Secret of square root module

The secret of the Sqrt module in the previous section is the algorithm for computing the square root.

6.3.2 Contracts: Preconditions and postconditions

The *precondition* of a function is what the caller (i.e. the client of the function) must ensure holds when calling the function. A precondition may specify the valid combinations of values of the arguments. It may also record any constraints on ant "global" state that the function accesses or modifies.

If the precondition holds, the supplier (i.e. developer) of the function must ensure that the function terminates with the *postcondition* satisfied. That is, the function returns the required values and/or alters the "global" state in the required manner.

We sometimes call the set of preconditions and postconditions for a function the *contract* for that function.

6.3.2.1 Contracts of square root module

In the Sqrt module defined in the previous section, the exported function sqrt' x has the precondition:

 $x \ge 0$

Function $sqrt{2}$ x is undefined for $x < 0$.

The postcondition of the function sqrt' x function is that the result returned is the correct mathematical value of the square root within the allowed tolerance. That is, for a tolerance of 0.001:

 $(sqrt x - 0.001)^2 < (sqrt x)^2 < (sqrt x)$

We can state preconditions and postconditions for the local functions sqrtIter, improve, average, and goodEnough in the Sqrt module. The preconditions for functions average and goodEnough are just the assertion True (i.e. always satisfied).

6.3.2.2 Contracts of Factorial module

Consider the factorial functions defined in Chapter 4. (These are in the source file [Factorial.hs](../Ch04/%3CFactorial.hs%3E).)

What are the preconditions and postconditions?

Functions fact1, fact2, and fact3 require that argument n be a natural number (nonnegative integer) value. If they are applied to a negative value for n, then the evaluation does not terminate. Operationally, they go into an "infinite loop" and likely will abort when the runtime stack overflows.

If function fact4 is called with a negative argument, then all guards and pattern matches fail. Thus the function aborts with a standard error message.

Similarly, function fact4' terminates with a custom error message for negative arguments.

Thus to ensure normal termination, we impose the precondition

 $n \geq 0$

on all these factorial functions.

The postcondition of all six factorial functions is that the result returned is the correct mathematical value of n factorial. For fact4, that is:

fact4 $n = fact'(n)$

None of the six factorial functions access or modify any global data structures, so we do not include other items in the precondition or postcondition.

Function fact5 is defined to be 1 for all arguments less than zero. So, if this is the desired result, we can weaken the precondition to allow all integer values, for example,

True

and strengthen the postcondition to give the results for negative arguments, for example:

fact5 $n = if n > = 0 then fact'(n) else 1$

6.3.3 Interfaces

It is important for information-hiding modules to have well-defined and stable interfaces.

According to Britton et al, an *interface* is a "set of assumptions ... each programmer needs to make about the other program . . . to demonstrate the correctness of his own program" [Britton 1981].

A module interface includes the type signatures (i.e. names, arguments, and return values), preconditions, and postconditions of all public operations (e.g. functions).

As we see in the next chapter, the interface also includes the *invariant* properties of the data values and structures manipulated by the module and the definitions of any new data types exported by the module. An invariant must be part of the precondition of public operations except operations that construct elements of the data type (i.e. constructors). It must also be part of the postcondition of public operations except operations that destroy elements of the data type (i.e. destructors).

As we have seen, in Haskell the module construct can be used to encapsulate an information-hiding module. The functions and data types exported form part of the interface to the module. One module can import another module, specifying its dependence on the interface of the other module.

Haskell's module facility and type system support part of what is needed to define an interface, but Haskell does not provide direct syntactic or semantic support for preconditions, postconditions, or invariant assertions.

6.3.3.1 Interface of square root module

The interface to the Sqrt module in the previous section consists of the function signature:

sqrt' :: Double -> Double

where sqrt' x has the precondition and postcondition defined above. None of the other functions are accessible outside the module Sqrt and, hence, are not part of the interface.

6.3.4 Abstract interfaces

An *abstract interface* is an interface that does not change when one module implementation is substituted for another [Britton 1981; Parnas 1978]. It concentrates on module's essential aspects and obscures incidental aspects that vary among implementations.

Information-hiding modules and abstract interfaces enable us to design and build software systems with multiple versions. The information-hiding approach seeks to identify aspects of a software design that might change from one version to another and to hide them within independent modules behind well-defined abstract interfaces.

We can reuse the software design across several similar systems. We can reuse an existing module implementation when appropriate. When we need a new implementation, we can create one by following the specification of the module's abstract interface.

6.3.4.1 Abstract interface of square root module

For the Sqrt example, if we implemented a different module with the same interface (signatures, preconditions, postconditions, etc.), then we could substitute the new module for Sqrt and get the same result.

In this case, the interface is an abstract interface for the set of module implementations.

Caveats: Of course, the time and space performance of the alternative modules might differ. Also, because of the nature of floating point arithmetic, it may be difficult to ensure both algorithms have precisely the same termination conditions.

6.3.5 Client-supplier relationship

The design and implementation of information-hiding modules must be approached from two points of view simultaneously:

supplier: the developers of the module—the providers of the services

client: the users of the module—the users of the services (e.g. the designers of other modules)

The *client-supplier relationship* is as represented in the following diagram:

(module user) (module)

The supplier's concerns include:

- efficient and reliable algorithms and data structures
- convenient implementation

• easy maintenance

The clients' concerns include:

- accomplishing their own tasks
- using the supplier module without effort to understand its internal details
- having a sufficient, but not overwhelming, set of operations.

As we have noted previously, the *interface* of a module is the set of features (i.e., public operations) provided by a supplier to clients.

A precise description of a supplier's interface forms a *contract* between clients and supplier.

The client-supplier contract:

1. gives the responsibilities of the client

These are the conditions under which the supplier must deliver results when the *preconditions* of the operations are satisfied (i.e. the operations are called correctly).

2. gives the responsibilities of the supplier

These are the benefits the supplier must deliver—make the *postconditions* hold at the end of the operation (i.e. the operations deliver the correct results).

The contract

- protects the client by specifying how much must be done by the supplier
- protects the supplier by specifying how little is acceptable to the client

If we are both the clients and suppliers in a design situation, we should consciously attempt to separate the two different areas of concern, switching back and forth between our supplier and client "hats".

6.3.6 Design criteria for interfaces

What else should we consider in designing a good interface for an informationhiding module?

In designing an interface for a module, we should also consider the following criteria. Of course, some of these criteria conflict with one another; a designer must carefully balance the criteria to achieve a good interface design.

Note: These are general principles; they are not limited to Haskell or functional programming. In object-oriented languages, these criteria apply to class interfaces.

- **Cohesion:** All operations must logically fit together to support a single, coherent purpose. The module should describe a single abstraction.
- **Simplicity:** Avoid needless features. The smaller the interface the easier it is to use the module.
- **No redundancy:** Avoid offering the same service in more than one way; eliminate redundant features.
- **Atomicity:** Do not combine several operations if they are needed individually. Keep independent features separate. All operations should be *primitive*, that is, not be decomposable into other operations also in the public interface.
- **Completeness:** All primitive operations that make sense for the abstraction should be supported by the module.
- **Consistency:** Provide a set of operations that are internally consistent in
	- **–** naming convention (e.g., in use of prefixes like "set" or "get", in capitalization, in use of verbs/nouns/adjectives),
	- **–** use of arguments and return values (e.g., order and type of arguments),
	- **–** behavior (i.e. make operations work similarly).

Avoid surprises and misunderstandings. Consistent interfaces make it easier to understand the rest of a system if part of it is already known.

The operations should be consistent with good practices for the specific language being used.

- **Reusability:** Do not customize modules to specific clients, but make them general enough to be reusable in other contexts.
- **Robustness with respect to modifications:** Design the interface of an module so that it remains stable even if the implementation of the module changes. (That is, it should be an abstract interface for an informationhiding module as we discussed above.)
- **Convenience:** Where appropriate, provide additional operations (e.g., beyond the complete primitive set) for the convenience of users of the module. Add convenience operations only for frequently used combinations after careful study.

We must trade off conflicts among the criteria. For example, we must balance:

- completeness versus simplicity
- reusability versus simplicity
- convenience versus consistency, simplicity, no redundancy, and atomicity

We must also balance these design criteria against efficiency and functionality.

6.4 What Next?

In this chapter, we considered modularity in the context of procedural abstraction. In the next chapter, we consider modularity in the context of data abstraction.

6.5 Exercises

- 1. State preconditions and postconditions for the following internal functions in the Sqrt module:
	- a. sqrtIter
	- b. improve
	- c. average
	- d. goodEnough
	- e. square
- 2. Develop recursive and iterative (looping) versions of the square root function from this chapter in an imperative language (e.g. Java, C++, Python 3, etc.)

6.6 Acknowledgements

In Summer and Fall 2016, I adapted and revised much of this work from my previous materials:

- Using Top-Down Stepwise Refinement (square root module) from my earlier implementations of this algorithm in Scala, Elixir, and Lua and from section 1.1.7 of Abelson and Sussman's *[Structure and Interpretation](http://mitpress.mit.edu/sicp/) [of Computer Programs](http://mitpress.mit.edu/sicp/)* [Abelson 1996]
- Modular Design and Programming from my Data Abstraction [Cunningham 2018a] and Modular Design [Cunningham 2018b] notes, which draw from the ideas of several references listed

In 2017, I continued to develop this work as sections 2.5-2.7 in Chapter 2, Basic Haskell Functional Programming), of my 2017 Haskell-based programming languages textbook.

In Spring and Summer 2018, I divided the previous Basic Haskell Functional Programming chapter into four chapters in the 2018 version of the textbook, now titled *Exploring Languages with Interpreters and Functional Programming.* Previous sections 2.1-2.3 became the basis for new Chapter 4, First Haskell Programs; previous Section 2.4 became Section 5.3 in the new Chapter 5, Types; and previous sections 2.5-2.7 were reorganized into new Chapter 6, Procedural Abstraction (this chapter), and Chapter 7, Data Abstraction. The discussion of

contracts for the factorial functions was moved from the 2017 Evaluation and Efficiency chapter to this chapter.

I maintain this chapter as text in Pandoc's dialect of Markdown using embedded LaTeX markup for the mathematical formulas and then translate the document to HTML, PDF, and other forms as needed.

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6.8 Terms and Concepts

Procedural abstraction, top-down stepwise refinement, abstract code, termination condition for recursion, Newton's method, Haskell module, module exports and imports, information hiding, module secret, encapsulation, precondition, postcondtion, contract, invariant, interface, abstract interface, design criteria for interfaces, software reuse, use of Haskell modules to implement informationhiding modules, client-supplier contract.