CSci 311, Models of Computation
Chapter 4
Pumming Lemma Outline and Example

Prepared by Eli Allen

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A few good tutorial videos:

Some of the videos use different notation, but the ideas are the same. If you don't understand the concepts from one example, hearing it explained a slightly different way can help.

- https://www.youtube.com/watch?v=LC0J45agGBU
- https://www.youtube.com/watch?v=BJQmV3oKqgo
- https://www.youtube.com/watch?v=oZcQQ5nHHXg

General outline of Pumping Lemma

Statement: I want to prove that some language L is not regular.

Proof by contradiction:

Assume L is regular.

Let m be some given number.

Let w be a string that is accepted by the language and is at least m characters long.

  that is, $|w| \geq m$

since $w \in L$ and $|w| \geq m$, the pumping lemma must apply. Specifically,

  $w = xyz$ where

$|y| \geq 1$ and $|xy| \leq m$

by the pumping lemma, the decomposition xy^iz of w, is also accepted L.

We try to find some value for i that results in a string not accepted by L. If we do, we have found a contradiction and proven that L is not regular.

Strategy

Given m, we should try to pick a string w that forces a predictable decomposition xyz.

> for example (Linz Example 4.9), if $L = \{w \in \Sigma^* : n_a(w) < n_b(w)\}$, then a good w to pick would be $a^m b^{m+1}$.
>
> > **This is the part that's hardest to understand**: because of the string that we picked, and we know that by the pumping lemma, $|xy| \leq m$, the only valid decomposition of our string must be so that xy consists entirely of a's.
> >
> > > For example if our string is a^3b^{3+1}, or $aaabbbb$, then because $m = 3$, and the length of xy cannot be greater than 3, we know that xy must consist entirely of a's. More specifically, y, which is the part of the string that will be pumped, must consist of only a's.
> >
> > So we have $y = a^k$ where $1 \leq k \leq m$.
>
> Then we have to pick an i so that the resulting string, w_i is not part of the language.
>
> > Going back to Linz Example 4.9, we pick $i = 2$ so that the resulting string $w_2 = a^{m+k}b^{m+1}$ is a contradiction because $m + k \geq m + 1$.
>
> In general there is no way to know if you should pump up ($i > 1$) or down ($i = 0$). You need to pick w so that you have a predictable decomposition xyz so that when you pump y^i, you know exactly what you will get, and it should be easy to find an xy^iz that violates L.

A Simple Example

$L = \{a^n b^n : n \in \mathbb{N}\}$

Statement: I want to prove L is not regualar.

Proof: Assume L is regular.

Given some m,

let $w = a^m b^m$

Since $w \in L$ and $|w| \geq m$, the pumping lemma applies to this selection of w.

So, by the pumping lemma, we get

$w = xyz$ where $|y| \geq 1$ and $|xy| \leq m$.

$y = a^k$ where $0 < k \leq m$

$x = a^q$ where $0 \leq q < m$

So z is everything that isn't in xy,

$z = a^{m-k-q} b^m$

choose $i = 2$,

$w_2 = xy^2 z = xyyz = a^q a^k a^k a^{m-k-q} b^m$

$= a^{q+k+k+m-k-q} b^m$

$= a^{m+k} b^m$, which is a contradiction because $m + k \neq m$