CSCI 311/500 Models of Computation

Homework 6

Due: Tuesday, November 27th at the beginning of class. No late homework assignments will be accepted.

Please follow the guidelines for writing up homework. Undergraduates do any FOUR of the five problems. Graduate students should do all five.

1. Construct an npda corresponding to the grammar:
   \[ S \rightarrow aABB \mid aAA \]
   \[ A \rightarrow aBB \mid a \]
   \[ B \rightarrow bBB \mid a \]

2. Describe the language accepted by the npda below. Give an English or set notation description.

3. Consider the npda \( M = (\{q_0, q_1, q_f\}, \{a, b\}, \{0, 1, z\}, \delta, q_0, z, \{q_f\}) \), where \( \delta \) is:
   \[ \delta(q_0, a, z) = \{(q_0, 11z)\} \]
   \[ \delta(q_0, a, 1) = \{(q_0, 111)\} \]
   \[ \delta(q_0, b, 1) = \{(q_1, \lambda)\} \]
   \[ \delta(q_1, b, 1) = \{(q_1, \lambda)\} \]
   \[ \delta(q_1, \lambda, z) = \{(q_f, z)\} \]

4. Construct an npda for the language \( L = \{a^n b^m c^{n+m} : n, m \geq 1\} \).

5. Consider the Turing Machine in Example 9.7 (reproduced below) for the language \( L = \{a^n b^n : n \geq 0\} \).
   Assume \( Q = \{q_0, q_1, q_2, q_3, q_4\} \), \( F = \{q_4\} \), \( \Sigma = \{a, b\} \), and \( \Gamma = \{a, b, x, y, \Delta\} \). Note that \( \Delta \) is used for the blank symbol.
   \[ 1 \ \delta(q_0, a) = (q_1, x, R) \]
   \[ 2 \ \delta(q_1, a) = (q_1, a, R) \]
   \[ 3 \ \delta(q_1, y) = (q_1, y, R) \]
   \[ 4 \ \delta(q_1, b) = (q_2, y, L) \]
   \[ 5 \ \delta(q_2, y) = (q_2, y, L) \]
   \[ 6 \ \delta(q_2, a) = (q_2, a, L) \]
   \[ 7 \ \delta(q_2, x) = (q_0, x, R) \]
   \[ 8 \ \delta(q_0, y) = (q_3, y, R) \]
9 \( \delta(q_3, y) = (q_3, y, R) \)
10 \( \delta(q_3, \Delta) = (q_4, \Delta, R) \)

Show the instantaneous descriptions using the Turing machine when presented with the following inputs. Label each transition with the corresponding number above.

(a) aba
(b) aaabbb