CSCI 311/500 Models of Computation
Homework 1

Due: Thursday, September 6th at Midnight

Please follow the guidelines for writing up homework.

1. Use set notation to define the following sets. Note: Do not use ellipsis (…) or just list the values in the set. In each case, assume Σ = {a, b}.
   (a) Strings that begin with “ab”.
   (b) Strings that have at least two symbols, but no more than seven.
   (c) Strings that contain the substring “abba”.
   (d) Strings with 0 or more a’s followed by 2 or more b’s followed by 0 or more a’s.

2. Describe the following languages in English as clearly and simply as possible. Assume Σ = {a, b}.
   (a) \( L = \{ww : w \in \Sigma^*\} \)
   (b) \( L = \{w : w \in \Sigma^+, n_a(w) \bmod 2 = 0\} \) where \( n_a(w) \) is the number of a’s in \( w \)
   (c) \( L = \{bvwb : v \in \Sigma^*, w \in \Sigma^*\} \)
   (d) \( L = \{awa : w \in \Sigma^*\} \cup \{bwb : w \in \Sigma^*\} \)

3. Given the grammar \( G = (\{S, A, B\}, \{a, b\}, S, P) \) where \( P \) is:
   \[
   S \rightarrow aA \mid bB \mid abS \mid \lambda \\
   A \rightarrow aA \mid \lambda \\
   B \rightarrow bB \mid \lambda
   \]
   (a) For each of the following strings, give a derivation, or indicate that it cannot be derived:
      i. aaa
      ii. ababa
      iii. bab
      iv. abbb
   (b) **Graduate students** Give an English description or set notation describing \( L(G) \).

4. Undergraduates write a grammar for two of the following three languages. **Graduates do all three.**
   (a) The set of strings that begin with “ab”.
   (b) \( L = \{w : w \in \Sigma^+, n_a(w) \bmod 2 = 0\} \)
   (c) \( L = \{awa : w \in \Sigma^*\} \cup \{bwb : w \in \Sigma^*\} \)